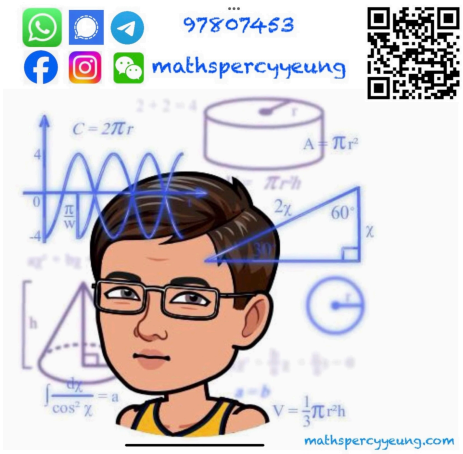

2022-2023 S6
1st TERM UT
MATH EP
M2

2022 – 2023
S6 First Term Uniform Test

MATHEMATICS Extended Part

Module 2 (Algebra and Calculus)

Question–Answer Book

3rd November, 2022
10:00 am – 11:00 am (1 hour)
This paper must be answered in English

INSTRUCTIONS

1. Write your name, class and class number in the spaces provided on this cover.

2. This paper consists of TWO sections, A and B.

3. Attempt ALL questions in this paper. Write your answers in the spaces provided in this Question – Answer Book. Do not write in the margins. Answers written in the margins will not be marked.

4. Unless otherwise specified, all working must be clearly shown.

5. Unless otherwise specified, numerical answers must be exact.

6. The diagrams in this paper are not necessarily drawn to scale.

Sections	Marks
A Total	**/15**
B Total	**/25**
TOTAL	**/40**

FORMULAS FOR REFERENCE

$$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$$

$$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$$

$$\tan(A \pm B) = \frac{\tan A \pm \tan B}{1 \mp \tan A \tan B}$$

$$2\sin A \cos B = \sin(A+B) + \sin(A-B)$$

$$2\cos A \cos B = \cos(A+B) + \cos(A-B)$$

$$2\sin A \sin B = \cos(A-B) - \cos(A+B)$$

$$\sin A + \sin B = 2\sin\frac{A+B}{2}\cos\frac{A-B}{2}$$

$$\sin A - \sin B = 2\cos\frac{A+B}{2}\sin\frac{A-B}{2}$$

$$\cos A + \cos B = 2\cos\frac{A+B}{2}\cos\frac{A-B}{2}$$

$$\cos A - \cos B = -2\sin\frac{A+B}{2}\sin\frac{A-B}{2}$$

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Section A (15 marks)

1. Evaluate $\lim_{x\to 0}\frac{e^{2x}-1}{x(e^x+3)}$. (3 marks)

Answers written in the margins will not be marked

2. (a) Find $\int (\ln 2x)^2 dx$.

(b) In Figure 1, the shaded region is bounded by the curve $y = \ln 2x$, the lines $y = 2x - 1$ and $x = 4$. Find the volume of the solid of revolution if the region is revolved about the x-axis.

(6 marks)

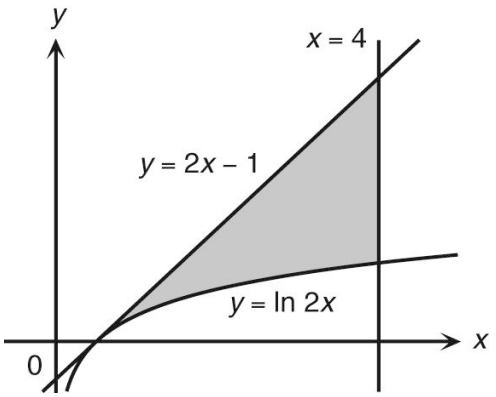

Figure 1

Answers written in the margins will not be marked

3. Figure 2 shows an inverted vessel in the shape of a right circular cone. The base radius and the height of the vessel are 6 cm and 15 cm respectively. Let V cm^3 and h cm be the volume and the depth of the water in the vessel respectively.

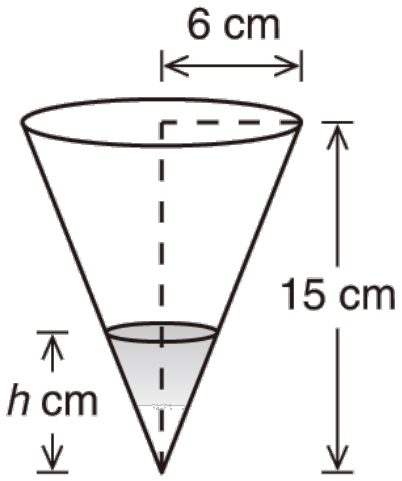

Figure 2

(a) Express V in terms of h.

(b) Water has been leaking out of the vessel through the apex for t min. The depth of the water is given by $h = \dfrac{15}{2e^{\frac{t}{4}} + 1}$. Find the rate of change of volume of the water in the vessel at $t = 4$. (Give your answer correct to 2 decimal places.)

(6 marks)

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Section B (25 marks)

4. Consider the following system of linear equations

$$(S): \begin{cases} x-2y-2z=k \\ x-hy+3z=0 \\ x-3y+hz=0 \end{cases} \quad \text{and} \quad (T): \begin{cases} 3x-6y-6z=-2 \\ x-hy+3z=0 \\ x-3y+hz=0 \\ 6x-5y-2z=h \end{cases}$$

(a) Assume (S) has a unique solution.

(i) Prove that $h^2 \neq 9$.

(ii) Solve (S).

(5 marks)

(b) For $h=-3$, find the value(s) of k for which (S) is consistent, and solve (S) for such value(s) of k.

(3 marks)

(c) If $h^2 \neq 9$, find the value(s) of h for which (T) is consistent.

Solve (T) for each of these values of h.

(4 marks)

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5. (a) (i) By considering $\tan[2(\frac{3\pi}{8})] = \tan\frac{3\pi}{4}$, show that $\tan\frac{3\pi}{8} = 1+\sqrt{2}$.

(ii) Suppose that $\tan u = 1-\sqrt{2}$. By (a)(i), find u, where $-\frac{\pi}{2} < u < \frac{\pi}{2}$.

(4 marks)

(b) (i) Express $x^2+\sqrt{2}x+1$ in the form of $(x+a)^2+b^2$, where a and b are constants.

(ii) Evaluate $\int_{-1}^{1} \frac{\sqrt{2}}{x^2+\sqrt{2}x+1}dx$.

(6 marks)

(c) Evaluate $\int_{-1}^{1} \frac{\sin\frac{3\pi}{4}}{x^2+2x\cos\frac{3\pi}{4}+1}dx$. (3 marks)

Answers written in the margins will not be marked

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END OF PAPER