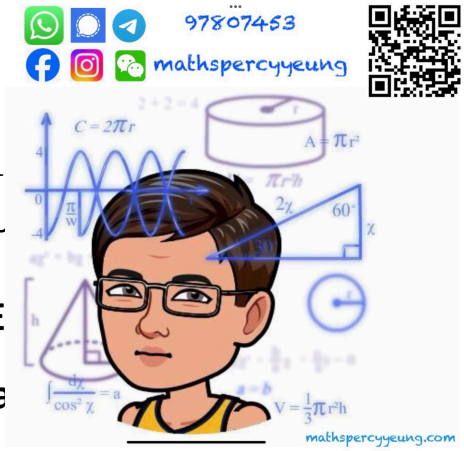
2021-2022 S6
1st TERM UT
MATH EP
M2



2021 –
S6 First Term U

MATHEMATICS E

Module 2 (Algebra

Question–Answer Book

4th November, 2021
10:00 am – 11:00 am (1 hour)
This paper must be answered in English

Sections	Marks
A Total	**/19**
B Total	**/21**
TOTAL	**/40**

INSTRUCTIONS

1. Write your name, class and class number in the spaces provided on this cover.

2. This paper consists of TWO sections, A and B.

3. Attempt ALL questions in this paper. Write your answers in the spaces provided in this Question – Answer Book. Do not write in the margins. Answers written in the margins will not be marked.

4. Unless otherwise specified, all working must be clearly shown.

5. Unless otherwise specified, numerical answers must be exact.

6. The diagrams in this paper are not necessarily drawn to scale.

FORMULAS FOR REFERENCE

$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$	$\sin A + \sin B = 2\sin\frac{A+B}{2}\cos\frac{A-B}{2}$
$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$	$\sin A - \sin B = 2\cos\frac{A+B}{2}\sin\frac{A-B}{2}$
$\tan(A \pm B) = \frac{\tan A \pm \tan B}{1 \mp \tan A \tan B}$	$\cos A + \cos B = 2\cos\frac{A+B}{2}\cos\frac{A-B}{2}$
$2\sin A \cos B = \sin(A+B) + \sin(A-B)$	$\cos A - \cos B = -2\sin\frac{A+B}{2}\sin\frac{A-B}{2}$
$2\cos A \cos B = \cos(A+B) + \cos(A-B)$	
$2\sin A \sin B = \cos(A-B) - \cos(A+B)$	

****

Section A (19 marks)

1. Let $A = \begin{pmatrix} a & -1 & 0 \\ -1 & a & 0 \\ 0 & 0 & a+1 \end{pmatrix}$, where a is a non-zero constant. Denote the 3×3 identity matrix by I.

(a) Find $\det(A - aI)$.

(b) It is given that $A^2 - a^2 I$ is a singular matrix. Using (a), find the value(s) of a.

(6 marks)

Answers written in the margins will not be marked

Answers written in the margins will not be marked

Answers written in the margins will not be marked

2. **(a)** Find $\int \sin^2 \theta \, d\theta$.

(b) Define $f(x) = 4x\left(1 - x^2\right)^{\frac{1}{4}}$ for all $x \in [0, 1]$. Denote the graph of $y = f(x)$ by G . Let R be the region bounded by G and the x-axis. Find the volume of the solid of revolution generated by revolving R about the x-axis.

(6 marks)

Answers written in the margins will not be marked

Answers written in the margins will not be marked

Answers written in the margins will not be marked

Answers written in the margins will not be marked

3. Consider the following system of homogeneous linear equations in real variables x, y and z.

$$(E): \begin{cases} x+3y+mz=0 \\ x+y+nz=0 \\ ny-mz=0 \end{cases}, \text{ where } m, n \in \mathbf{R}.$$

Assume that (E) has non-trivial solutions.

(a) Show that $n^2-mn-2m=0$.

(b) Suppose $m=-9$.

(i) Find the values of n.

(ii) For each of the values of n found in (b) (i), solve (E).

(7 marks)

Answers written in the margins will not be marked

Answers written in the margins will not be marked

Answers written in the margins will not be marked

Section B (21 marks)

4. ABC is a triangle. D is the mid-point of AC. E is a point lying on BC such that $BE : EC = 1 : r$. AB produced and DE produced meet at the point F. It is given that $DE : EF = 1 : 10$.
Let $\overrightarrow{OA} = 2\mathbf{i} + 3\mathbf{j} - 2\mathbf{k}$, $\overrightarrow{OB} = 4\mathbf{i} + 4\mathbf{j} - \mathbf{k}$ and $\overrightarrow{OC} = 8\mathbf{i} - 3\mathbf{j} - 2\mathbf{k}$, where O is the origin.

(a) By expressing $\overrightarrow{AE}$ and $\overrightarrow{AF}$ in terms of r, find r. (4 marks)

(b) **(i)** Find $\overrightarrow{AD} \bullet \overrightarrow{DE}$.

(ii) Are B, D, C and F concyclic? Explain your answer.

(5 marks)

Answers written in the margins will not be marked

Answers written in the margins will not be marked

Answers written in the margins will not be marked

5. Let $\overrightarrow{OA} = \mathbf{i} - 2\mathbf{j} + 3\mathbf{k}$, $\overrightarrow{OB} = -5\mathbf{i} + 4\mathbf{j} - \mathbf{k}$, $\overrightarrow{OC} = -3\mathbf{i} + 2\mathbf{j} - 5\mathbf{k}$ and $\overrightarrow{OD} = -4\mathbf{i} - \mathbf{j} - 2\mathbf{k}$, where O is the origin. Denote the plane containing A, B and C by Π. Let P be the projection of D on Π.

(a) Find

(i) $\overrightarrow{AB} \times \overrightarrow{AC}$,

(ii) $\overrightarrow{PD}$.

(4 marks)

(b) Let Q be a point lying on AB such that DQ is perpendicular to AB.

(i) Find $\overrightarrow{DQ}$.

(ii) Show that $\overrightarrow{PQ}$ and $\overrightarrow{AB}$ are orthogonal.

(5 marks)

(c) Find the angle between $\triangle ABD$ and Π. (3 marks)

Answers written in the margins will not be marked

Answers written in the margins will not be marked

Answers written in the margins will not be marked

Answers written in the margins will not be marked

Answers written in the margins will not be marked

END OF PAPER