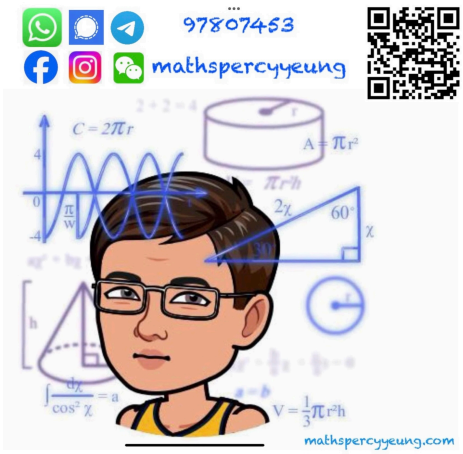
2020-21 F.6
MATH EP
M2



2020 - 2021 1st Term Uniform Test

MATHEMATICS Extended Part
Module 2 (Algebra and Calculus)

Question-Answer Book

2nd November, 2020. (Monday)

10:15 am – 11:15 am (1 hour)

This paper must be answered in English.

INSTRUCTIONS

1. After the announcement of the start of the examination, you should first write your name, class and class number in the spaces provided on this cover.
2. This paper consists of Section A and Section B.
3. Answer ALL questions. Write your answers in the spaces provided in this Question-Answer Book.
4. Graph paper and supplementary answer sheets will be supplied on request. Write your name, class, class number and mark the question number box on each sheet.
5. Unless otherwise specified, all working must be clearly shown.
6. Unless otherwise specified, numerical answers must be exact.
7. In this paper, vectors may be represented by bold-type letters such as **u**, but candidates are expected to use appropriate symbols such as $\vec{u}$ in their working.
8. The diagrams in this paper are not necessarily drawn to scale.

	Marks
Section A	/ 15
Section B	/ 25
Grand Total	/ 40

Section A (15 marks)

1. In **Figure 1**, it is given that the coordinates of A , B and C are $(-1, 2, 1)$, $(0, 1, 2)$ and $(1, 0, -1)$ respectively. N is a point on AC such that $BN \perp AC$.

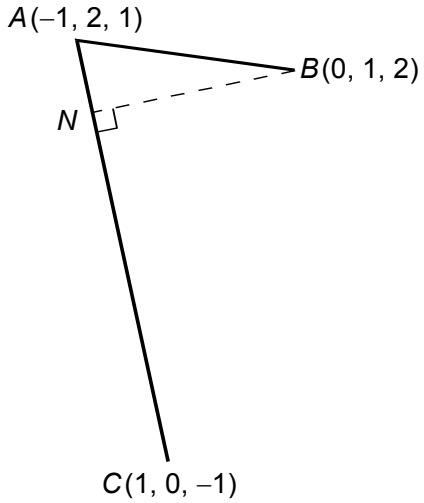


Figure 1

(a) Find $\overrightarrow{AN}$.

(4 marks)

(b) Find the shortest distance from B to AC .

(2 marks)

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2. Let $\mathbf{m}$ and $\mathbf{n}$ be two vectors, where $|\mathbf{m}| = 3$, $|\mathbf{n}| = 2$ and the angle between them is $120°$.

It is given that $\mathbf{a} = \mathbf{m} - 4\mathbf{n}$ and $\mathbf{b} = 2\mathbf{m} + \mathbf{n}$.

(a) Find $\mathbf{m} \cdot \mathbf{n}$.

(2 marks)

(b) Find $|\mathbf{a}|$ and $|\mathbf{b}|$.

(3 marks)

(c) **(i)** Find the angle between $\mathbf{a}$ and $\mathbf{b}$.

(ii) Find the area of the parallelogram with **a** and **b** as two adjacent sides.

(Give your answers correct to 3 significant figures.)

(4 marks)

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Section B (25 marks)

3. Let $\overrightarrow{OP} = \mathbf{i} + \mathbf{j} + 4\mathbf{k}$ and $\overrightarrow{OQ} = 5\mathbf{i} - 7\mathbf{j} - 4\mathbf{k}$, where O is the origin. R is a point lying on PQ such that $PR : RQ = 1 : 3$.

(a) Find $\overrightarrow{OP} \times \overrightarrow{OR}$.

(3 marks)

(b) Define $\overrightarrow{OS} = \overrightarrow{OP} + \overrightarrow{OR}$. Find the area of the quadrilateral $OPSR$.

(2 marks)

(c) Let N be a point such that $\overrightarrow{ON} = \lambda\left(\overrightarrow{OP} \times \overrightarrow{OR}\right)$, where λ is a real number.

(i) Is $\overrightarrow{NR}$ perpendicular to $\overrightarrow{PQ}$? Explain your answer.

(ii) **(1)** Prove that $\overrightarrow{OR}$ is perpendicular to $\overrightarrow{PQ}$.

(2) It is given that $\lambda = \frac{2}{9}$. Denote the angle between ΔOPQ and ΔNPQ by θ. Find $\tan\theta$.

(7 marks)

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4. Let $\overrightarrow{OA} = -\mathbf{i} + \mathbf{j} - \mathbf{k}$, $\overrightarrow{OB} = 3\mathbf{i} + \mu\mathbf{j} + \lambda\mathbf{k}$ and $\overrightarrow{OC} = \mathbf{j} + 2\mathbf{k}$, where O is the origin, and λ and μ are real numbers. It is given that $\angle AOB = \angle BOC = 90°$.

(a) Find the values of λ and μ.

(3 marks)

(b) **(i)** Find the volume of the tetrahedron $OABC$.

(ii) If ΔABC is the base, find the height of the tetrahedron.

(iii) Find the angle between the line OB and the plane ABC, correct to the nearest $0.1°$.

(7 marks)

(c) Let D be the foot of perpendicular from O to the plane ABC. Sam claims that D lies on the line joining B and the mid-point of AC. Do you agree? Explain your answer.

(3 marks)

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