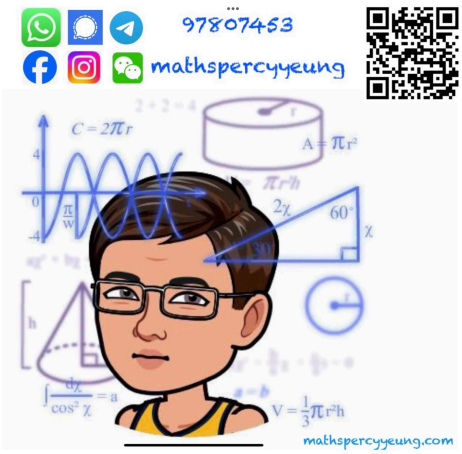
2020-21
MATH EP
F.5 M2



2020 - 2021 1st Term Examination

Form 5 MATHEMATICS
Extended Part
Module 2 (Algebra and Calculus)

Question–Answer Book

14th January, 2021. (Thursday)

8:15 am – 10:15 am (2 hours)

This paper must be answered in English.

INSTRUCTIONS

1. After the announcement of the start of the examination, you should first write your name, class and class number in the spaces provided on this cover.
2. This paper consists of Section A and Section B.
3. Answer ALL questions. Write your answers in the spaces provided in this Question-Answer Book.
4. Graph paper and supplementary answer sheets will be supplied on request. Write your name, class, class number and mark the question number box on each sheet.
5. Unless otherwise specified, all working must be clearly shown.
6. Unless otherwise specified, numerical answers must be exact.
7. The diagrams in this paper are not necessarily drawn to scale.

	Marks
Section A	/ 44
Section B	/ 38
Grand Total	/ 82

FORMULAS FOR REFERENCE

$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$	$\sin A + \sin B = 2\sin\frac{A+B}{2}\cos\frac{A-B}{2}$
$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$	$\sin A - \sin B = 2\cos\frac{A+B}{2}\sin\frac{A-B}{2}$
$\tan(A \pm B) = \frac{\tan A \pm \tan B}{1 \mp \tan A \tan B}$	$\cos A + \cos B = 2\cos\frac{A+B}{2}\cos\frac{A-B}{2}$
$2\sin A \cos B = \sin(A+B) + \sin(A-B)$	$\cos A - \cos B = -2\sin\frac{A+B}{2}\sin\frac{A-B}{2}$
$2\cos A \cos B = \cos(A+B) + \cos(A-B)$	
$2\sin A \sin B = \cos(A-B) - \cos(A+B)$	

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Section A (44 marks)

1. Evaluate $\lim\limits_{x\to 0}\frac{e^{2x}-1}{\sin 3x}$. (3 marks)

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2. Find $\frac{dy}{dx}$ for $y = \sqrt{3x-1}$ from first principles. (4 marks)

Answers written in the margins will not be marked.

3. It is given that $(1-px)^n = 1+\frac{5}{4}x+10p^2x^2+$ terms involving higher powers of x, where n is a positive integer.

(a) Find the values of n and p.

(b) Hence, find the coefficient of x^3 in the expansion.

(6 marks)

Answers written in the margins will not be marked.

4. Prove that $\dfrac{2\sin 2x+\sin 4x}{2\sin 2x-\sin 4x}=\cot^2 x$. (3 marks)

Answers written in the margins will not be marked.

5. It is given that $\sin\frac{A}{2}=3\sin\frac{B}{2}$.

(a) Show that $\dfrac{\sin\frac{A}{2}-\sin\frac{B}{2}}{\sin\frac{A}{2}+\sin\frac{B}{2}}=\frac{1}{2}$.

(b) Hence, show that $\tan\frac{A-B}{4}=\frac{1}{2}\tan\frac{A+B}{4}$. (4 marks)

Answers written in the margins will not be marked.

6. It is given that $P(-1, 0)$ is a point on $y^2 = x^2 - x\sin y - 1$. Find the equation of the tangent to the curve at P. (4 marks)

7. Let $x = y^2 + e^{3y}$. Find $\dfrac{d^2y}{dx^2}$ in terms of y. (3 marks)

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8. **(a)** Prove, by mathematical induction, that

$$1\times 3\times 5+2\times 4\times 6+3\times 5\times 7+\cdots+n(n+2)(n+4)=\frac{n(n+1)(n+4)(n+5)}{4}$$

for all positive integers n.

(b) Find the value of $31\times 33\times 35+32\times 34\times 36+33\times 35\times 37+\cdots+51\times 53\times 55$.

(7 marks)

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9. The slope at any point (x, y) of a curve is given by $\frac{dy}{dx} = 2e^{2x} - 3e^{x}$. The y-intercept of the curve is 0. Find the area bounded by the curve and the x-axis. (5 marks)

Answers written in the margins will not be marked.

10. **(a)** Express $-2+4x-x^2$ in the form of $a(x-h)^2+k$.

(b) Hence, find $\int_1^3 \sqrt{-2+4x-x^2}\,dx$.

(5 marks)

Answers written in the margins will not be marked.

Section B (38 marks)

11. Let $f(x) = \dfrac{x^3}{x^2 - 3}$, where $x \neq \pm\sqrt{3}$.

(a) Find the x- and y-intercept(s) of the graph of $y = f(x)$. (1 mark)

(b) Find $f'(x)$ and $f''(x)$. (3 marks)

(c) Find the maximum/minimum point(s) and point(s) of inflexion of the curve $y = f(x)$. (6 marks)

(d) Find the asymptote(s) of the curve. (2 marks)

(e) Sketch the curve $y = f(x)$. (3 marks)

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12. (a) Figure 1 shows a shaded region enclosed by the curve $x = \sqrt{y \ln y}$, the line $y = k$ and the y-axis. A vessel is formed by revolving the shaded region about the y-axis. Show that the volume of the vessel is $\frac{\pi}{4}\left(2k^2 \ln k - k^2 + 1\right)$ cubic units.

(5 marks)

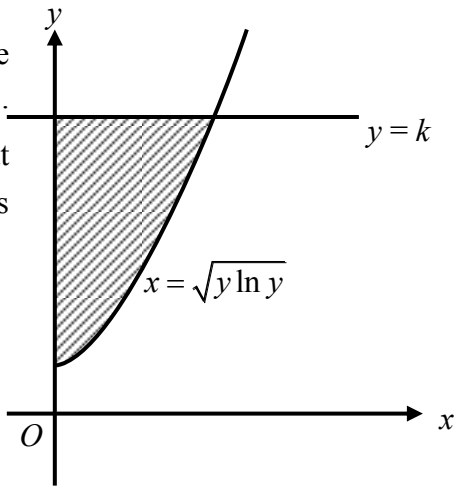

Figure 1

(b) In Figure 2, vessels M and N are vessels formed as in **(a)**. Initially, vessel N is empty and vessel M contains some water. There is a small hole at the bottom of vessel M such that water is leaking through the hole into vessel N. When the depths of water in vessels M and N are $(e^2 - 1)$ units and $(e - 1)$ units respectively, the water level in vessel M decreases at a rate of 1 unit per second. Find the rate of increase of water level in vessel N.

(5 marks)

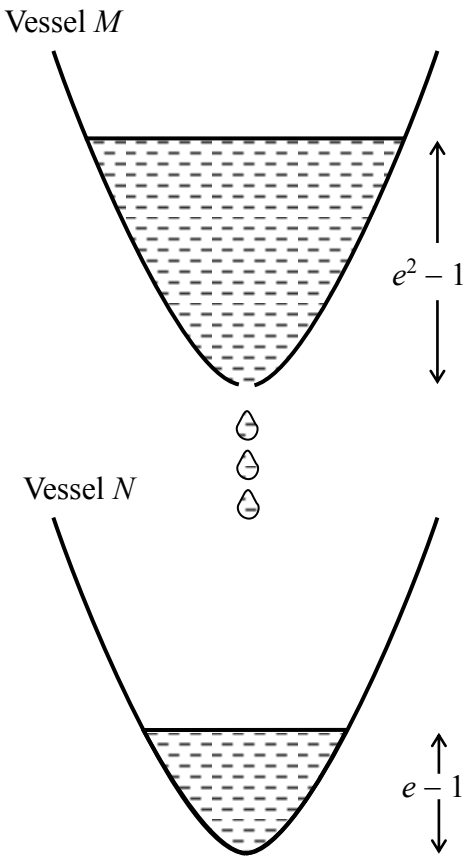

Figure 2

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13. (a) Using integration by parts, or otherwise, find $\int xe^x dx$ and $\int x^2 e^x dx$. (3 marks)

(b) Show that $\int_0^x f(t)dt = -\int_0^{-x} f(-t)dt$.

Hence, deduce that if $f(x)$ is an even function, then the function

$$g(x) = \int_0^x f(t)dt$$

is an odd function.

(3 marks)

(c) Let $h(x)$ be a differentiable function.

Suppose that $h(0) = 10$ and $h'(x) = x^2 e^x + \sqrt{x^2+1}\cos^2 x$.

(i) Show that

$$h(x) = (x^2 - 2x + 2)e^x + 8 + k(x)$$

for some odd function $k(x)$.

(ii) Using the above results, or otherwise, evaluate $\int_{-1}^{1} h(x)dx$

(7 marks)

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