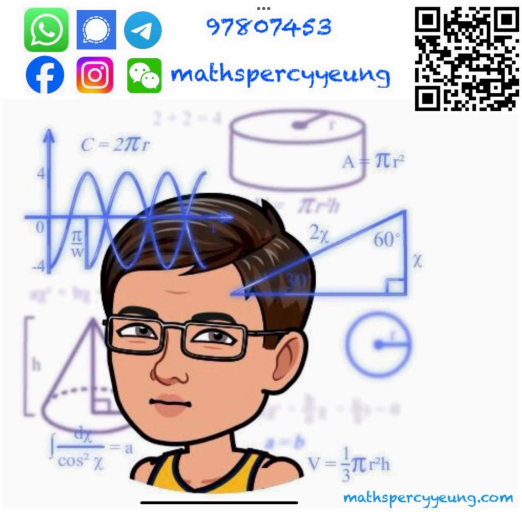
2019-20
MATH EP
F.5 M2



2019 - 2020 1st Term Examination

Form 5 MATHEMATICS
Extended Part
Module 2 (Algebra and Calculus)

Question–Answer Book

8th January, 2020. (Wednesday)

10:15 am – 12:15 pm (2 hours)

This paper must be answered in English.

INSTRUCTIONS

1. After the announcement of the start of the examination, you should first write your name, class and class number in the spaces provided on this cover.
2. This paper consists of Section A and Section B.
3. Answer ALL questions. Write your answers in the spaces provided in this Question-Answer Book.
4. Graph paper and supplementary answer sheets will be supplied on request. Write your name, class, class number and mark the question number box on each sheet.
5. Unless otherwise specified, all working must be clearly shown.
6. Unless otherwise specified, numerical answers must be exact.

	Marks
Section A	/ 44
Section B	/ 36
Grand Total	/ 80

FORMULAS FOR REFERENCE

$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$	$\sin A + \sin B = 2\sin\frac{A+B}{2}\cos\frac{A-B}{2}$
$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$	$\sin A - \sin B = 2\cos\frac{A+B}{2}\sin\frac{A-B}{2}$
$\tan(A \pm B) = \frac{\tan A \pm \tan B}{1 \mp \tan A \tan B}$	$\cos A + \cos B = 2\cos\frac{A+B}{2}\cos\frac{A-B}{2}$
$2\sin A \cos B = \sin(A+B) + \sin(A-B)$	$\cos A - \cos B = -2\sin\frac{A+B}{2}\sin\frac{A-B}{2}$
$2\cos A \cos B = \cos(A+B) + \cos(A-B)$	
$2\sin A \sin B = \cos(A-B) - \cos(A+B)$	

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Section A (44 marks)

1. **(a)** Evaluate $\lim\limits_{x \to 0} \dfrac{e^{5x}\sin 2x}{e^{3x} - e^{-3x}}$.

(b) Evaluate $\lim\limits_{x \to \infty} x \ln\left(1 + \dfrac{2}{x} - \dfrac{3}{x^2}\right)$.

(5 marks)

Answers written in the margins will not be marked.

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2. Let n be a positive integer. In the expansion of $(1+3x)^n\left(x-\frac{4}{x}\right)^2$, the constant term is 5176.

(a) Find the value of n.

(b) Find the coefficient of x^2 in the expansion.

(5 marks)

Answers written in the margins will not be marked.

3. **(a)** Using mathematical induction, prove that $\sum_{k=n}^{2n} k^2 = \frac{n(n+1)(14n+1)}{6}$ for all positive integers n.

(b) Using (a), evaluate $\sum_{k=25}^{100} k^2$.

(7 marks)

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4. **(a)** If $\tan A - 3\cot B = 0$, prove that $\cos(A-B) + 2\cos(A+B) = 0$.

(b) Hence solve the equation $\tan(x+29°) = 3\cot(31° - x)$, where $90° < x < 180°$.

(5 marks)

Answers written in the margins will not be marked.

5. Let $f(x) = 3 + \dfrac{x^2}{x-4}$.

(a) Find $f'(x)$.

(b) Find the maximum point(s) and the minimum point(s) of the graph of $y = f(x)$.

(c) Find all the asymptotes of the graph of $y = f(x)$.

(8 marks)

Answers written in the margins will not be marked.

6. **(a)** Using integration by parts, find $\int e^x \cos(\pi x)\, dx$.

(b) Using integration by substitution, evaluate $\int_0^2 e^{2-x} \cos(\pi x)\, dx$.

(7 marks)

Answers written in the margins will not be marked.

7. **(a)** Using integration by parts, find $\int \frac{\ln x}{x^2}\,dx$.

(b) For $x > 0$, the slope of the normal at any point (x, y) of curve C is given by $\frac{x^2}{\ln x - e}$.

It is given that C passes through the point $(1, 1)$. Find the equation of C .

(7 marks)

Answers written in the margins will not be marked.

Section B (36 marks)

8. Figure 1 shows a rectangular wooden frame $PQRS$, where $PQ = 1$ m and $QR = 2$ m. QD is a thin rod of length 1 m hinged at Q. When the end point D of the rod moves away from P inside the frame, F is a point on PS such that $FD \perp PS$. It is given that all the points in the figure are on the same horizontal plane.

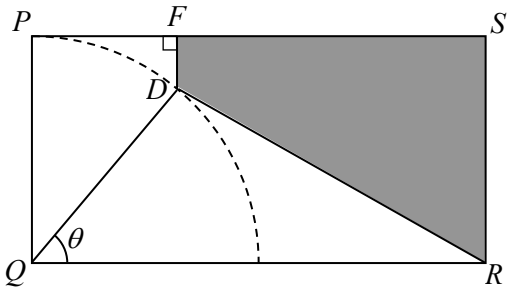

Figure 1

Let $\angle DQR = \theta$ and A m^2 be the area of the trapezium $FDRS$, where $0 < \theta < \frac{\pi}{2}$.

(a) **(i)** Express A in terms of θ.

(ii) Show that $\frac{dA}{d\theta} = \frac{1}{2}(\sin\theta - \cos\theta)(2 - \sin\theta - \cos\theta)$ for $0 < \theta < \frac{\pi}{2}$.

(3 marks)

(b) **(i)** Show that $2 - \sin\theta - \cos\theta > 0$ for $0 < \theta < \frac{\pi}{2}$.

(ii) Hence, find the minimum value of A.

(6 marks)

(c) It is given that θ decreases at a constant rate of $\frac{1}{10}$ rad/s. When $\theta = \frac{\pi}{3}$, find the rate of change of the area of the trapezium $FDRS$.

(3 marks)

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9. Consider the curve Γ: $y=\frac{1}{3}\sqrt{12-x^2}$, where $0<x<2\sqrt{3}$. Denote the tangent to Γ at $x=3$ by L .

(a) Find the equation of L .

(3 marks)

(b) Let C be the curve $y=\sqrt{4-x^2}$, where $0<x<2$. It is given that L is a tangent to C .

(i) Find the point(s) of contact of L and C .

(ii) Find the point(s) of intersection of C and Γ.

(iii) Find the area of the region bounded by L, C and Γ.

(9 marks)

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10. **(a)** Show that $\sin\theta+\cos\theta=\sqrt{2}\cos\left(\frac{\pi}{4}-\theta\right)$.

(2 marks)

(b) Let $g(x)$ be a continuous function. Show that $\int_0^{\frac{\pi}{4}} g(\cos x)\,dx=\int_0^{\frac{\pi}{4}} g\left[\cos\left(\frac{\pi}{4}-x\right)\right]\,dx$.

(3 marks)

(c) Using **(a)** and **(b)**, evaluate $\int_0^{\frac{\pi}{4}} \ln(1+\tan\theta)\,d\theta$.

(4 marks)

(d) Evaluate $\int_0^1 \frac{\ln(1+x)^2}{1+x^2}\,dx$.

(3 marks)

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