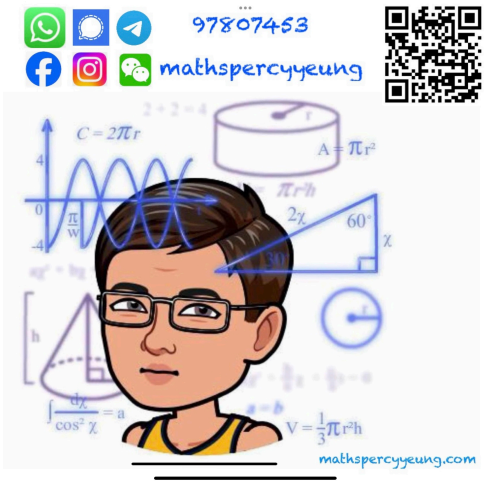
2018-19 F.5
MATH EP
M2



2018 - 2019 Form 5 Second Term Uniform Test

MATHEMATICS Extended Part Module 2 (Algebra and Calculus)

Question–Answer Book

28th March, 2019. (Thursday)

9:30 am – 10:30 am (1 hour)

This paper must be answered in English.

Section	Marks
A	**/ 27**
B	**/ 13**
TOTAL	**/40**

INSTRUCTIONS

1. After the announcement of the start of the examination, you should first write your name, class and class number in the spaces provided on this cover.
2. This paper consists of Section A and Section B.
3. Answer ALL questions. Write your answers in the spaces provided in this Question-Answer Book.
4. Graph paper and supplementary answer sheets will be supplied on request. Write your name, class, class number and mark the question number box on each sheet.
5. Unless otherwise specified, all working must be clearly shown.
6. Unless otherwise specified, numerical answers must be exact.
7. In this paper, vectors may be represented by bold-type letters such as **u**, but candidates are expected to use appropriate symbols such as $\vec{u}$ in their working.
8. The diagrams in this paper are not necessarily drawn to scale.

FORMULAS FOR REFERENCE

$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$	$\sin A + \sin B = 2\sin\dfrac{A+B}{2}\cos\dfrac{A-B}{2}$
$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$	$\sin A - \sin B = 2\cos\dfrac{A+B}{2}\sin\dfrac{A-B}{2}$
$\tan(A \pm B) = \dfrac{\tan A \pm \tan B}{1 \mp \tan A \tan B}$	$\cos A + \cos B = 2\cos\dfrac{A+B}{2}\cos\dfrac{A-B}{2}$
$2\sin A \cos B = \sin(A+B) + \sin(A-B)$	$\cos A - \cos B = -2\sin\dfrac{A+B}{2}\sin\dfrac{A-B}{2}$
$2\cos A \cos B = \cos(A+B) + \cos(A-B)$	
$2\sin A \sin B = \cos(A-B) - \cos(A+B)$	

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Section A (27 marks)

1. In Figure 1, the shaded region is bounded by the curve $y = x^3$, the x-axis, the lines $y = 1$ and $x = -1$. A solid is generated by revolving the region about the line $x = -1$. Find the volume of the solid of revolution. (4 marks)

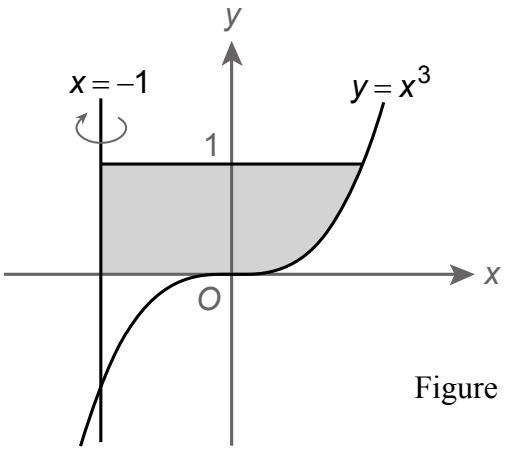

Figure 1

Answers written in the margins will not be marked.

2. Consider the system of linear equations $(E)\begin{cases} kx - z = 1 \\ (k+2)x - y + 2z = -1, \\ x + ky - z = 1 \end{cases}$ where k is a real number.

(a) Show that the system (E) has a unique solution for all values of k.

(b) Solve the system (E) by Cramer's rule in terms of k.

(5 marks)

Answers written in the margins will not be marked.

3. Find

(a) $\int \sin^3 x \cos x dx$.

(b) $\int x^2 e^x dx$.

(5 marks)

Answers written in the margins will not be marked.

4. Consider the homogeneous linear system $(*)\begin{cases} 2x+(a-1)y-z=0 \\ ax+3y-4z=0 \\ -2x+2y+z=0 \end{cases}$, where a is a real number.

(a) Find the values of a such that (*) has non-trivial solutions.

(b) According to the values of a obtained in (a), solve (*).

(7 marks)

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5.

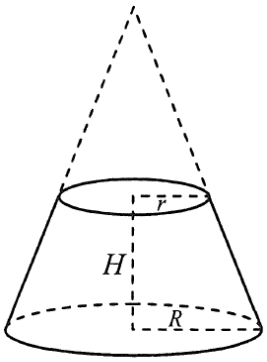

Figure 2

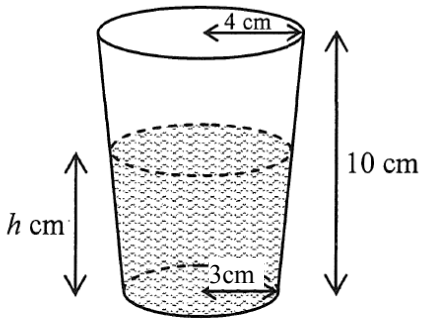

Figure 3

A frustum of height H is made by cutting off a right circular cone of base radius r from a right circular cone of base radius R (see Figure 2). It is given that the volume of the frustum is $\frac{\pi}{3}H\left(r^2+rR+R^2\right)$.

An empty glass is in the form of an inverted frustum described above with height 10 cm, the radii of the rim and the base 4 cm and 3 cm respectively. Water is being poured into the glass. Let h cm $(0 \le h \le 10)$ be the depth of the water inside the glass at time t s (see Figure 3).

(a) Show that the volume V cm^3 of water inside the glass at time t s is given by

$$V=\frac{\pi}{300}\left(h^3+90h^2+2700h\right) .$$

(b) If the volume of water in the glass is increasing at the rate 7π cm^{3}s^{-1}, find the rate of increase of depth of water at the instant when $h = 5$.

(6 marks)

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Section B (13 marks)

6. (a) Solve the equation

$$\begin{vmatrix} 1-x & 4 \\ 2 & 3-x \end{vmatrix} = 0 \quad \ldots\ldots (*) .$$

(2 marks)

(b) Let x_1 , x_2 ($x_1 < x_2$) be the roots of (*) . Let $P = \begin{pmatrix} a & c \\ b & 1 \end{pmatrix}$. It is given that

$$\begin{pmatrix} 1 & 4 \\ 2 & 3 \end{pmatrix}\begin{pmatrix} a \\ b \end{pmatrix} = x_1 \begin{pmatrix} a \\ b \end{pmatrix} , \quad \begin{pmatrix} 1 & 4 \\ 2 & 3 \end{pmatrix}\begin{pmatrix} c \\ 1 \end{pmatrix} = x_2 \begin{pmatrix} c \\ 1 \end{pmatrix} \text{ and } \quad |P| = 1 ,$$

where a , b and c are constants.

(i) Find P .

(ii) Evaluate $P^{-1}\begin{pmatrix} 1 & 4 \\ 2 & 3 \end{pmatrix}P$.

(iii) Using (b)(ii), evaluate $\begin{pmatrix} 1 & 4 \\ 2 & 3 \end{pmatrix}^{12}$.

(11 marks)

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