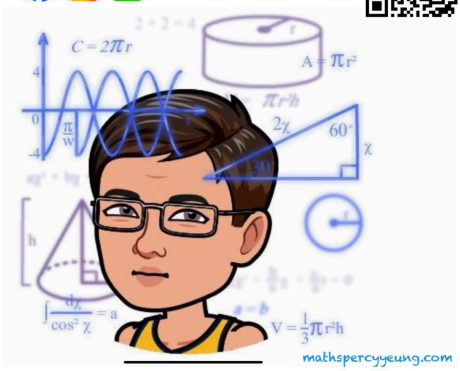

2018-19 F5
MATH EP
M2

Form 5 2018 - 2019 2nd Term Examination

MATHEMATICS Extended Part
Module 2 (Algebra and Calculus)

Question–Answer Book

14th June, 2019. (Friday)

8:15 am – 10:45 am (2.5 hours)

This paper must be answered in English.

INSTRUCTIONS

1. After the announcement of the start of the examination, you should first write your name, class and class number in the spaces provided on this cover.
2. This paper consists of Section A and Section B.
3. Answer ALL questions. Write your answers in the spaces provided in this Question-Answer Book.
4. Graph paper and supplementary answer sheets will be supplied on request. Write your name, class, class number and mark the question number box on each sheet.
5. Unless otherwise specified, all working must be clearly shown.
6. Unless otherwise specified, numerical answers must be exact.
7. The diagrams in this paper are not necessarily drawn to scale.

Section A Question No.	Marks
1	
2	
3	
4	
5	
6	
7	
8	
9	
Section A Total	**/ 50**

Section B Question No.	Marks
10	
11	
12	
13	
Section B Total	**/ 50**

Grand Total	**/ 100**

FORMULAS FOR REFERENCE

$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$	$\sin A + \sin B = 2\sin\frac{A+B}{2}\cos\frac{A-B}{2}$
$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$	$\sin A - \sin B = 2\cos\frac{A+B}{2}\sin\frac{A-B}{2}$
$\tan(A \pm B) = \frac{\tan A \pm \tan B}{1 \mp \tan A \tan B}$	$\cos A + \cos B = 2\cos\frac{A+B}{2}\cos\frac{A-B}{2}$
$2\sin A \cos B = \sin(A+B) + \sin(A-B)$	$\cos A - \cos B = -2\sin\frac{A+B}{2}\sin\frac{A-B}{2}$
$2\cos A \cos B = \cos(A+B) + \cos(A-B)$	
$2\sin A \sin B = \cos(A-B) - \cos(A+B)$	

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Section A (50 marks)

1. Expand $(x+3)^5$. Hence, find the difference between the coefficient of x^3 and the constant term in the expansion of $(x+3)^5\left(x-\frac{4}{x}\right)^2$. (5 marks)

Answers written in the margins will not be marked.

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2. By mathematical induction, prove that $1-2+4-8+...+(-2)^{n-1}=\frac{1-(-2)^n}{3}$ for all positive integers n. (5 marks)

Answers written in the margins will not be marked.

3. Evaluate the following limits.

(a) $\lim\limits_{x\to 0}\dfrac{\sin 2x}{\tan 3x}$ (b) $\lim\limits_{x\to+\infty}\left(\sqrt{x^2+x+1}-\sqrt{x^2+1}\right)$ (6 marks)

Answers written in the margins will not be marked.

4. Let $f(x) = x\cos 2x$. Express $f\left(\frac{\pi}{4}+h\right)$ in terms of h. Hence find $f'\left(\frac{\pi}{4}\right)$ from first principles.

(4 marks)

Answers written in the margins will not be marked.

5. Find the equation of the normal to the curve $x^2 + xy = 6y^3$ at (−8, 2). (5 marks)

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6. (a) If $\tan A - 3\cot B = 0$, prove that $\cos(A-B) + 2\cos(A+B) = 0$.

(b) Hence solve the equation $\tan(x+29°) = 3\cot(31° - x)$, where $90° < x < 180°$. (5 marks)

Answers written in the margins will not be marked.

7. In Figure 1, the shaded region is bounded by the curves $x = y^2$ and $x = \frac{y^2}{4} + 3$. The curves intersect at A and B.

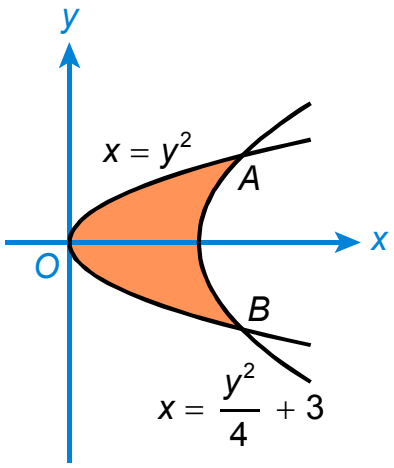

Figure 1

(a) Find the coordinates of A and B.

(b) Find the volume of the solid of revolution generated by revolving the shaded region about the straight line AB.

(6 marks)

Answers written in the margins will not be marked.

8. Let $\overrightarrow{OA} = 2\,\mathbf{i}$ and $\overrightarrow{OB} = \mathbf{i} + 2\,\mathbf{j}$. M is the mid-point of OA and N lies on AB such that $BN : NA = k : 1$. BM intersects ON at P (see Figure 2).

(a) Express $\overrightarrow{ON}$ in terms of k.

(b) If A, N, P and M are concyclic, find the value of k.

(5 marks)

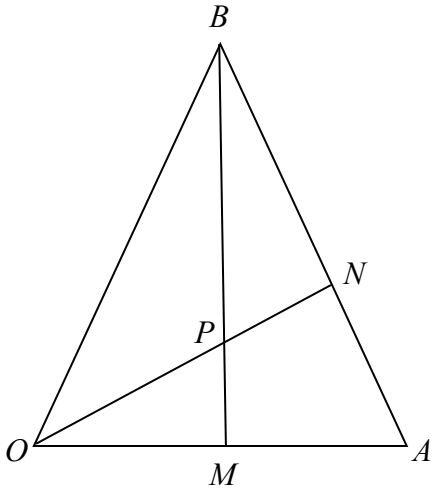

Figure 2

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9. In Figure 3, the radius of circle C is r units. C is revolved about the y-axis to form a spherical container. Water is poured into the container and its depth is h units.

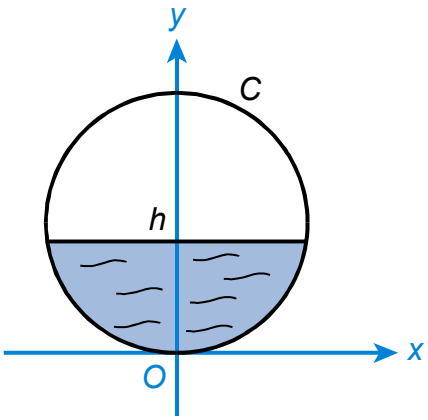

Figure 3

(a) Express the equation of C in terms of r. (1 mark)

Assume that $r = 10$.

(b) If the volume of water inside the container is V cubic units, express V in terms of h. (4 marks)

(c) (i) Find $\frac{dV}{dh}$ when $h = 5$.

(ii) It is given that water is poured at a rate of 10 cubic units per second. When the depth of water is 5 units, find the rate of increase of water level. (4 marks)

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Section B (50 marks)

10. Let $P=\begin{pmatrix}4 & 0 & 1\\ 2 & 3 & 2\\ 1 & 0 & 4\end{pmatrix}$, $Q=\begin{pmatrix}1 & 1 & 0\\ 2 & 0 & 1\\ 1 & -1 & 0\end{pmatrix}$ and $R=Q^{-1}PQ$.

(a) Find Q^{-1}. (2 marks)

(b) Show that $R=\begin{pmatrix}5 & 0 & 0\\ 0 & 3 & 0\\ 0 & 0 & 3\end{pmatrix}$. (2 marks)

(c) (i) Find P^n.

(ii) Is P^n-R^n a singular matrix? Explain your answer. (7 marks)

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11. Consider the system of linear equations in x, y and z:

$$(E)\quad \begin{cases} x+2y-z=b \\ 2x+(a-2)y+3z=0 \\ ax-3y+4z=b^2 \end{cases}$$, where a and b are real numbers

(a) Assume that $b = 0$. Show that (E) has non-trivial solutions if and only if $(a+4)^2=25$.

(3 marks)

(b) Assume that $b \neq 0$.

(i) If (E) has a unique solution, find the values of a and express y in terms of a and b.

(ii) Find the value(s) of b such that (E) has no solution.

(7 marks)

(c) Determine whether there exists an integral solution of the system of linear equations

$\begin{cases} x+2y-z=-1 \\ 2x-y+3z=0 \\ x-3y+4z=1 \end{cases}$ which satisfies the equation $-5x+5y+2z=11$.

(4 marks)

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12. (a) Evaluate $\int_0^1 \frac{du}{3+u^2}$. (3 marks)

(b) (i) Show that $\int_{\frac{a}{2}}^{a} f(x)dx = \int_0^{\frac{a}{2}} f(a-x)dx$.

(ii) Hence show that $\int_0^a f(x)dx = \int_0^{\frac{a}{2}} \left[f(x) + f(a-x) \right] dx$.

(3 marks)

(c) It is given that $\frac{1-x^2}{3+x^2} \equiv \frac{A}{3+x^2} + B$, find the values of A and B. (2 marks)

(d) Using (a), (b)(ii) and (c), evaluate $\int_0^{\pi} \frac{x \sin^3 x}{3+\cos^2 x} dx$. (4 marks)

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13. The position vectors of the points A, B, C and D are $\mathbf{i}-\mathbf{j}+\mathbf{k}$, $2\mathbf{i}-3\mathbf{j}+\mathbf{k}$, $-\mathbf{i}+2\mathbf{j}-\mathbf{k}$ and $2\mathbf{i}-2\mathbf{j}+5\mathbf{k}$ respectively. Let E be the projection of D on the plane which contains A, B and C.

(a) Find

(i) $\overrightarrow{AB}\times\overrightarrow{AC}$,

(ii) the volume of the parallelepiped formed by $\overrightarrow{AB}$, $\overrightarrow{AC}$ and $\overrightarrow{AD}$,

(iii) $\overrightarrow{DE}$.

(5 marks)

(b) Let F be the foot of perpendicular from D to AB.

(i) Find $\overrightarrow{DF}$.

(ii) Are A, E and F collinear? Explain your answer.

(iii) Someone claims that the area of ΔADF is greater than 5. Do you agree? Explain your answer.

(8 marks)

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