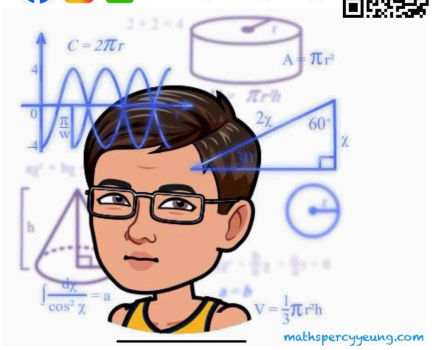
2018-19 F.5
MATH EP
M2



Form 5 2018 - 2019 1st Term Examination

MATHEMATICS Extended Part
Module 2 (Algebra and Calculus)

Question–Answer Book

8th January, 2019. (Tuesday)

10:15 am – 12:15 pm (2 hours)

This paper must be answered in English.

INSTRUCTIONS

1. After the announcement of the start of the examination, you should first write your name, class and class number in the spaces provided on this cover.
2. This paper consists of Section A and Section B.
3. Answer ALL questions. Write your answers in the spaces provided in this Question-Answer Book.
4. Graph paper and supplementary answer sheets will be supplied on request. Write your name, class, class number and mark the question number box on each sheet.
5. Unless otherwise specified, all working must be clearly shown.
6. Unless otherwise specified, numerical answers must be exact.
7. The diagrams in this paper are not necessarily drawn to scale.

Section A Question No.	Marks
Section A Total	**/ 40**

Section B Question No.	Marks
9	
10	
11	
Section B Total	**/ 40**

Grand Total	**/ 80**

FORMULAS FOR REFERENCE

$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$	$\sin A + \sin B = 2\sin\frac{A+B}{2}\cos\frac{A-B}{2}$
$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$	$\sin A - \sin B = 2\cos\frac{A+B}{2}\sin\frac{A-B}{2}$
$\tan(A \pm B) = \frac{\tan A \pm \tan B}{1 \mp \tan A \tan B}$	$\cos A + \cos B = 2\cos\frac{A+B}{2}\cos\frac{A-B}{2}$
$2\sin A \cos B = \sin(A+B) + \sin(A-B)$	$\cos A - \cos B = -2\sin\frac{A+B}{2}\sin\frac{A-B}{2}$
$2\cos A \cos B = \cos(A+B) + \cos(A-B)$	
$2\sin A \sin B = \cos(A-B) - \cos(A+B)$	

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Section A (40 marks)

1. Prove that $\frac{1}{\sqrt{x}} - \frac{1}{\sqrt{x+h}} = \frac{h}{(x+h)\sqrt{x} + x\sqrt{x+h}}$. Hence, find $\frac{d}{dx}\sqrt{\frac{3}{x}}$ from first principles.

(5 marks)

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2. Let $T_n = \frac{n-1}{2^n}$. By mathematical induction, prove that $T_1 + T_2 + T_3 + \cdots + T_n = 1 - \frac{n+1}{2^n}$ for all positive integers n. (5 marks)

3. Find the limit.

(a) $\lim_{x \to 0} \frac{\sin 2x}{\tan 3x}$. **(b)** $\lim_{x \to 0} \frac{1-e^{-x}}{x}$ (5 marks)

4. In the expansion of $(1-x)(a+bx)^8$, the coefficient of x^5 is zero. Find the value of $\frac{a}{b}$.

(5 marks)

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5. It is given that $2x+\sqrt{x^2+xy}=8$. Find $\dfrac{dy}{dx}$ when $x=2$. (4 marks)

6. Find the following integrals.

(a) $\int (2x-3)^{2018}dx$ **(b)** $\int \cos^3 x\sin^5 x dx$ (5 marks)

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7. It is given that α and β are two acute angles.

(a) Show that $\dfrac{\sin\alpha+\sin\beta}{\cos\alpha+\cos\beta}=\tan(\dfrac{\alpha+\beta}{2})$.

(b) If $3\sin\alpha-4\cos\alpha=4\cos\beta-3\sin\beta$, find the value of $\tan(\alpha+\beta)$. (5 marks)

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8. **(a)** Find $\int xe^{-x}dx$.

(b)

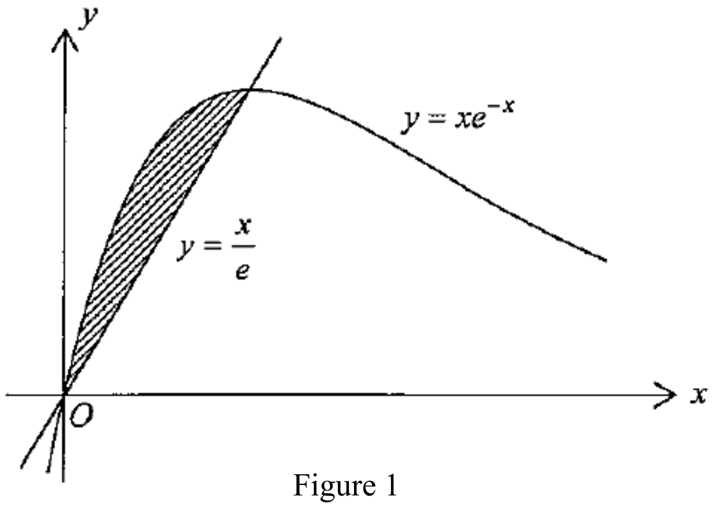

Figure 1

Figure 1 shows the shaded region bounded by the curve $y = xe^{-x}$ and the straight line $y = \frac{x}{e}$. Find the area of the shaded region. (6 marks)

Section B (40 marks)

9. In Figure 2, a straight line L touches a circle of centre O at the point P. AOB is a diameter of the circle and $OA = 4$ cm. D is a point on L such that AD is perpendicular to L. Let $AD = x$ cm $(0 < x < 8)$ and S cm^2 be the area of ΔADP.

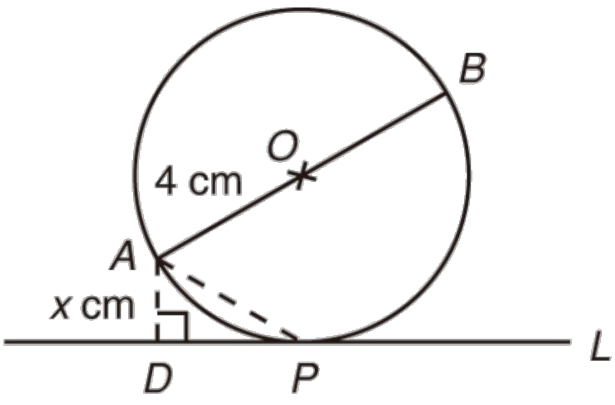

Figure 2

(a) **(i)** Express DP in terms of x.

(ii) Express S in terms of x. (3 marks)

(b) Find the maximum value of S. (5 marks)

(c) When $AD = 2$ cm, its length decreases at a rate of $\frac{1}{\sqrt{3}}$ cm/s. At that moment, find the rate of change of the area of ΔADP. (3 marks)

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10. **(a)** Let $f(x)$ be a continuous function defined on the interval [0, a], where a is a positive constant. Prove that $\int_0^a f(x)dx = \int_0^a f(a-x)dx$. (3 marks)

(b) Prove that $\int_0^{\frac{\pi}{4}} \ln(1+\tan x)dx = \int_0^{\frac{\pi}{4}} \ln\left(\frac{2}{1+\tan x}\right)dx$. (3 marks)

(c) Using (b), prove that $\int_0^{\frac{\pi}{4}} \ln(1+\tan x)dx = \frac{\pi \ln 2}{8}$. (3 marks)

(d) Using integration by parts, evaluate $\int_0^{\frac{\pi}{4}} \frac{x\sec^2 x}{1+\tan x}dx$. (3 marks)

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11. Let $f(x) = \dfrac{2x}{1+x^2}$.

(a) **(i)** Find $f'(x)$ and $f''(x)$.

(ii) For the curve $y = f(x)$, find all the extreme points and points of inflexion.

(8 marks)

(b) Find all the asymptote(s) of the curve $y = f(x)$. (1 mark)

(c) Sketch the curve $y = f(x)$. (2 marks)

(d) Let R be the region bounded by the x-axis , the curve $y = f(x)$ and the line $x = a$, where a is the x-coordinate of the maximum point of the curve $y = f(x)$.

(i) Find the area of R.

(ii) If R is revolved about the x-axis, find the volume of the solid of revolution.

(6 marks)

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END OF PAPER

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