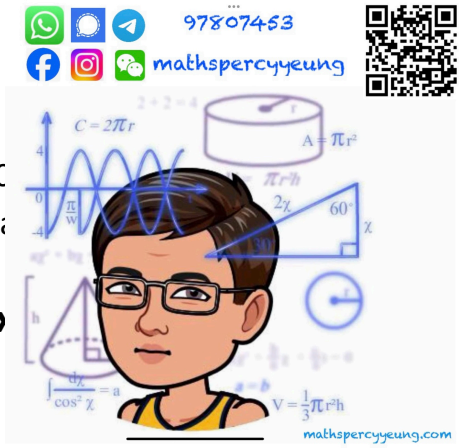
2022-2023 S4
1st TERM EXAM
MATH EP
M2



2022 – 20
S4 First Term Exa

MATHEMATICS Ex

Module 2 (Algebra

Question–Answer Book

5th January, 2023
8:15 am – 9:30 am (1 hour 15 minutes)
This paper must be answered in English

Section	Marks
A Total	**/34**
B Total	**/16**
TOTAL	**/50**

INSTRUCTIONS

1. Write your name, class and class number in the spaces provided on this cover.

2. This paper consists of TWO sections, A and B.

3. Attempt ALL questions in this paper. Write your answers in the spaces provided in this Question-Answer Book. Do not write in the margins. Answers written in the margins will not be marked.

4. Unless otherwise specified, all working must be clearly shown.

5. Unless otherwise specified, numerical answers must be exact.

6. The diagrams in this paper are not necessarily drawn to scale.

FORMULAS FOR REFERENCE

$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$	$\sin A + \sin B = 2\sin\frac{A+B}{2}\cos\frac{A-B}{2}$
$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$	$\sin A - \sin B = 2\cos\frac{A+B}{2}\cos\frac{A-B}{2}$
$\tan(A \pm B) = \frac{\tan A \pm \tan B}{1 \mp \tan A \tan B}$	$\cos A + \cos B = 2\cos\frac{A+B}{2}\cos\frac{A-B}{2}$
$2\sin A \cos B = \sin(A+B) + \sin(A-B)$	$\cos A - \cos B = -2\sin\frac{A+B}{2}\sin\frac{A-B}{2}$
$2\cos A \cos B = \cos(A+B) + \cos(A-B)$	
$2\sin A \sin B = \cos(A-B) - \cos(A+B)$	

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Section A (34 marks)

1. **(a)** It is given that $\frac{1}{n(n+1)} \equiv \frac{A}{n} + \frac{B}{n+1}$, where A and B are constants. Find the values of A and B.

(b) Hence, find the value of $\sum_{k=3}^{110} \frac{1}{k(k+1)}$.

(4 marks)

Answers written in the margins will not be marked

2. **(a)** Expand $(3x-1)^7$ in ascending powers of x up to the term in x^3.

(b) Hence find the coefficient of x^3 in the expansion of $(x+2)(3x-1)^7$.

(4 marks)

3. When $\left(3x^5+\frac{1}{x^4}\right)^n$ is expanded in descending powers of x, the 6th term of the expansion is a constant.

(a) Find n.

(b) Find the constant term of the expansion.

(4 marks)

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4. Using mathematical induction, prove that $\frac{1\cdot 2^2}{2\cdot 3}+\frac{2\cdot 2^3}{3\cdot 4}+\frac{3\cdot 2^4}{4\cdot 5}+\cdots+\frac{n\cdot 2^{n+1}}{(n+1)(n+2)}=\frac{2^{n+2}}{n+2}-2$ for all positive integers n. (5 marks)

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5. Prove that $\dfrac{\cos^2 A - \cos^2 B}{(\sin A - \sin B)^2} = -\tan\left(\dfrac{A+B}{2}\right)\cot\left(\dfrac{A-B}{2}\right)$. (3 marks)

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6. **(a)** Prove the identity $\tan 2x = \dfrac{2\tan x}{2-\sec^2 x}$.

(b) Using **(a)**, prove the identity $\cot y = \dfrac{8\cot 8y}{(2-\sec^2 4y)(2-\sec^2 2y)(2-\sec^2 y)}$.

(5 marks)

Answers written in the margins will not be marked

7. Evaluate each of the following.

(a) $\lim\limits_{x \to -2} \dfrac{x+2}{x+\sqrt{3x+10}}$

(b) $\lim\limits_{x \to +\infty} \dfrac{\sqrt{2-4x+3x^2}}{3+6x}$

(c) $\lim\limits_{x \to 0} \dfrac{4x \cot 2x}{\cos x}$

(9 marks)

Answers written in the margins will not be marked

Section B (16 marks)

8. **(a)** Using mathematical induction, prove that $\sum_{r=1}^{n} r(r+4) = \frac{1}{6}n(n+1)(2n+13)$ for all positive integers n. (5 marks)

(b) It is given that $1+2+3+\ldots+n = \frac{n(n+1)}{2}$ for all positive integers n. Evaluate $1\cdot 6+2\cdot 7+3\cdot 8+\ldots+40\cdot 45$. (3 marks)

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9. **(a)** Prove that $1-\sin^4\theta-\cos^4\theta=\frac{1}{2}\sin^2 2\theta$. (2 marks)

(b) By **(a)**, show that $\sin\frac{\pi}{8}$ is a root of $8x^4-8x^2+1=0$. (3 marks)

(c) Hence, or otherwise, express $\sin\frac{\pi}{8}$ in surd form. (3 marks)

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