2021-2022 S4
1st TERM EXAM
MATH EP
M2

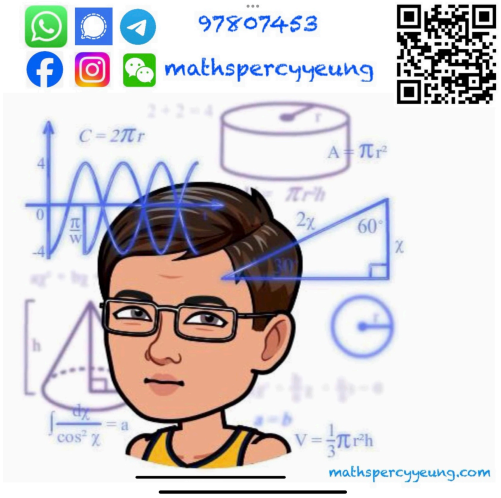
2021 – 2022
S4 First Term Examination

MATHEMATICS Extended Part

Module 2 (Algebra and Calculus)

Question–Answer Book

12th January, 2022
8:15 am – 9:30 am (1 hour 15 minutes)
This paper must be answered in English



Section	Marks
A Total	**/33**
B Total	**/17**
TOTAL	**/50**

INSTRUCTIONS

1. Write your name, class and class number in the spaces provided on this cover.

2. This paper consists of TWO sections, A and B.

3. Attempt ALL questions in this paper. Write your answers in the spaces provided in this Question-Answer Book. Do not write in the margins. Answers written in the margins will not be marked.

4. Unless otherwise specified, all working must be clearly shown.

5. Unless otherwise specified, numerical answers must be exact.

6. The diagrams in this paper are not necessarily drawn to scale.

FORMULAS FOR REFERENCE

$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$	$\sin A + \sin B = 2\sin\frac{A+B}{2}\cos\frac{A-B}{2}$
$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$	$\sin A - \sin B = 2\cos\frac{A+B}{2}\sin\frac{A-B}{2}$
$\tan(A \pm B) = \frac{\tan A \pm \tan B}{1 \mp \tan A \tan B}$	$\cos A + \cos B = 2\cos\frac{A+B}{2}\cos\frac{A-B}{2}$
$2\sin A \cos B = \sin(A+B) + \sin(A-B)$	$\cos A - \cos B = -2\sin\frac{A+B}{2}\sin\frac{A-B}{2}$
$2\cos A \cos B = \cos(A+B) + \cos(A-B)$	
$2\sin A \sin B = \cos(A-B) - \cos(A+B)$	

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Section A (33 marks)

1. Prove that $\frac{\sin x + \sin 2x + \sin 3x}{\cos x + \cos 2x + \cos 3x} = \tan 2x$. (3 marks)

Answers written in the margins will not be marked

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2. Prove that $\sin\left(\frac{3\pi}{2}-\theta\right)\sec\left(\frac{\pi}{2}+\theta\right)-\cos\left(\frac{3\pi}{2}+\theta\right)\csc\left(\frac{\pi}{2}-\theta\right)=2\cot 2\theta.$ (5 marks)

3. Let $\sum_{i=1}^{4} x_i^{\,2}=21,\ \sum_{i=1}^{5} x_i y_i=12$ and $\sum_{i=1}^{5} y_i^{\,2}=11$.

(a) It is given that $x_5=2$. Find the value of $\sum_{i=1}^{5} x_i^{\,2}$.

(b) Find the value of $\sum_{i=1}^{5}(x_i-2y_i)^2$.

(4 marks)

Answers written in the margins will not be marked

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4. If the constant term in the expansion of $\left(kx+\frac{4}{x^2}\right)^9$ is 84, where k is a constant, find the values of k. (3 marks)

Answers written in the margins will not be marked

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5. The sum of the coefficients of x and x^2 in the expansion of $(1+2x)^n$ is 98, where n is a positive integer.

(a) Find the value of n.

(b) Find the constant term in the expansion of $\left(1+\frac{2}{x}-\frac{3}{x^2}\right)(1+2x)^n$.

(5 marks)

Answers written in the margins will not be marked

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6. Prove, by mathematical induction, that $6 + 16 + 40 + \ldots + (n + 2)(2^n) = -2 + (n + 1)(2^{n+1})$ for all positive integers n. (5 marks)

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7. Evaluate each of the following.

(a) $\lim\limits_{x \to 6} \dfrac{\sqrt{x^2 + 13} - 7}{x - 6}$

(b) $\lim\limits_{x \to \infty} \dfrac{(3x + 1)(x - 6)(2x + 7)}{(4x^2 + 5)(9x - 8)}$

(c) $\lim\limits_{x \to 0} \dfrac{e^{3x} - 1}{\sin 4x}$

(8 marks)

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Section B (17 marks)

8. **(a)** Prove, by mathematical induction, that for all positive integers n,

$$\frac{1}{2\times 4}+\frac{1}{3\times 5}+\cdots+\frac{1}{(n+1)(n+3)}=\frac{n(5n+13)}{12(n+2)(n+3)}\ .$$

(5 marks)

(b) Using **(a)**, evaluate each of the following expressions.

(i) $\dfrac{1}{2\times 4}+\dfrac{1}{3\times 5}+\cdots+\dfrac{1}{47\times 49}$

(ii) $\displaystyle\sum_{r=23}^{48}\left(\frac{50}{r+1}\times\frac{51}{r+3}\right)$

(4 marks)

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9. Consider the equation $\cos 3\theta + \sin 2\theta = 0 \ldots (*)$.

(a) Show that $\frac{3\pi}{10}$ is a root of $(*)$. (2 marks)

(b) Show that $\cos 3\theta = 4\cos^3\theta - 3\cos\theta$. (3 marks)

(c) Find the value of $\sin\frac{3\pi}{10}$. (3 marks)

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END OF PAPER

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