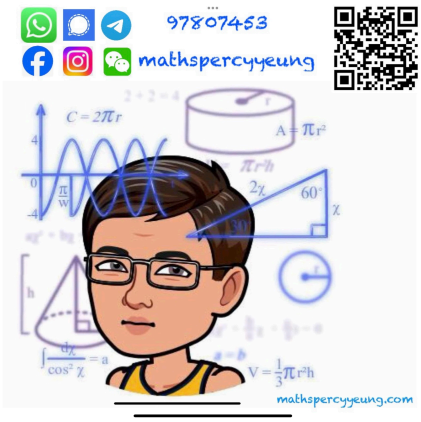
2018-19
MATH EP
M2



2018 - 2019 2nd Term Examination

MATHEMATICS Extended Part
Module 2 (Algebra and Calculus)

Question–Answer Book

6th June, 2019. (Thursday)

11:00 am – 12:30 pm (1.5 hours)

This paper must be answered in English.

INSTRUCTIONS

1. After the announcement of the start of the examination, you should first write your name, class and class number in the spaces provided on this cover.
2. This paper consists of Section A and Section B.
3. Answer ALL questions. Write your answers in the spaces provided in this Question-Answer Book.
4. Graph paper and supplementary answer sheets will be supplied on request. Write your name, class, class number and mark the question number box on each sheet.
5. Unless otherwise specified, all working must be clearly shown.
6. Unless otherwise specified, numerical answers must be exact.

	Marks
Section A	**/ 50**
Section B	**/ 12**
Grand Total	**/ 62**

FORMULAS FOR REFERENCE

$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$	$\sin A + \sin B = 2\sin\frac{A+B}{2}\cos\frac{A-B}{2}$
$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$	$\sin A - \sin B = 2\cos\frac{A+B}{2}\sin\frac{A-B}{2}$
$\tan(A \pm B) = \frac{\tan A \pm \tan B}{1 \mp \tan A \tan B}$	$\cos A + \cos B = 2\cos\frac{A+B}{2}\cos\frac{A-B}{2}$
$2\sin A \cos B = \sin(A+B) + \sin(A-B)$	$\cos A - \cos B = -2\sin\frac{A+B}{2}\sin\frac{A-B}{2}$
$2\cos A \cos B = \cos(A+B) + \cos(A-B)$	
$2\sin A \sin B = \cos(A-B) - \cos(A+B)$	

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Section A (50 marks)

1. Find $\frac{d}{dx}(\tan 5x)$ from first principles. (4 marks)

Answers written in the margins will not be marked.

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2. It is given that $(1+kx)^n = 1+33x+495x^2$ + terms involving higher powers of x . If k is a non-zero real number and n is a positive integer, find the values of k and n . (5 marks)

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3. **(a)** Prove, by mathematical induction, that for all positive integers n,

$$\frac{4}{2}+\frac{5}{4}+\frac{6}{8}+\cdots+\frac{n+3}{2^n}=5-\frac{n+5}{2^n} \quad .$$

(b) Hence simplify $\dfrac{2n+4}{2^{2n+1}}+\dfrac{2n+5}{2^{2n+2}}+\dfrac{2n+6}{2^{2n+3}}+\cdots+\dfrac{3(n+1)}{8^n}$.

(8 marks)

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4. **(a)** Simplify $f(\theta)=\dfrac{\csc\theta\left(\sec^2\theta-1\right)}{\tan(\pi+\theta)\sin\left(\dfrac{\pi}{2}-\theta\right)}$.

(b) If $\tan\theta=-\sqrt{2}$, find the value of $f(\theta)$.

(5 marks)

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5. **(a)** Show that $\cos 2\theta = \dfrac{1-\tan^2\theta}{1+\tan^2\theta}$.

(b) Using **(a)** , deduce that $\tan^2 67.5° = 3+2\sqrt{2}$.

(5 marks)

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6. Evaluate the following limits.

(a) $\lim_{x \to \infty} \left(\sqrt{4x^2 + x} - 2x \right)$

(b) $\lim_{x \to 0} \frac{x\left(e^{4x} - 1\right)}{\cos^2 3x - 1}$

(c) $\lim_{x \to \infty} x \ln\left(\frac{x+3}{x-2}\right)$

(8 marks)

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7. Differentiate each of the following functions with respect to x.

(a) $y = x^2 \sqrt{2x+1}$

(b) $y = \ln\left(\dfrac{\sin x}{x^2}\right)$

(6 marks)

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8. If $6x^2 - xy + y^3 = 3$, find $\left.\dfrac{dy}{dx}\right|_{(2,-3)}$.

(4 marks)

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9. It is given that k is a constant. If $y = e^{2x} \sin x$ satisfies the equation $\frac{d^2 y}{dx^2} - 4\frac{dy}{dx} + ky = 0$ for all real values of x, find the value of k.

(5 marks)

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Section B (12 marks)

10. Let $f(x)=\dfrac{x^2+bx+18}{x+a}$, where a and b are non-zero constants and $x\neq -a$. It is given that the graph of $y=f(x)$ has a vertical asymptote $x=-2$ and an oblique asymptote $y=x+7$.

(a) Find the values of a and b.

(3 marks)

(b) **(i)** Find the x-intercept and the y-intercept of the graph of $y=f(x)$.

(ii) Find the extreme point(s) of the graph of $y=f(x)$.

(iii) Does the graph of $y=f(x)$ have any point(s) of inflexion? Explain your answer.

(7 marks)

(c) Sketch the graph of $y=f(x)$.

(2 marks)

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END OF PAPER

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