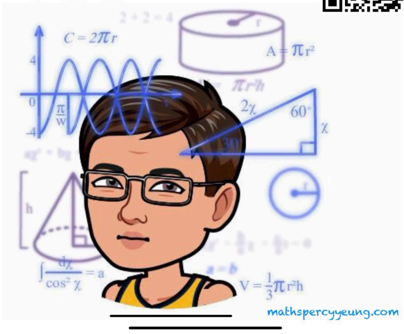
2018-19 F.4
MATH EP
M2



Form 4 2018 - 2019 1st Term Examination

MATHEMATICS Extended Part
Module 2 (Algebra and Calculus)

Question–Answer Book

10th January, 2019. (Thursday)

11:00 am – 12:15 pm (1 hour 15 minutes)

This paper must be answered in English.

INSTRUCTIONS

1. After the announcement of the start of the examination, you should first write your name, class and class number in the spaces provided on this cover.
2. This paper consists of Section A and Section B.
3. Answer ALL questions. Write your answers in the spaces provided in this Question-Answer Book.
4. Graph paper and supplementary answer sheets will be supplied on request. Write your name, class, class number and mark the question number box on each sheet.
5. Unless otherwise specified, all working must be clearly shown.
6. Unless otherwise specified, numerical answers must be exact.

	Marks
Section A	/ 39
Section B	/ 11
Grand Total	/ 50

FORMULAS FOR REFERENCE

$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$	$\sin A + \sin B = 2\sin\frac{A+B}{2}\cos\frac{A-B}{2}$
$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$	$\sin A - \sin B = 2\cos\frac{A+B}{2}\sin\frac{A-B}{2}$
$\tan(A \pm B) = \frac{\tan A \pm \tan B}{1 \mp \tan A \tan B}$	$\cos A + \cos B = 2\cos\frac{A+B}{2}\cos\frac{A-B}{2}$
$2\sin A \cos B = \sin(A+B) + \sin(A-B)$	$\cos A - \cos B = -2\sin\frac{A+B}{2}\sin\frac{A-B}{2}$
$2\cos A \cos B = \cos(A+B) + \cos(A-B)$	
$2\sin A \sin B = \cos(A-B) - \cos(A+B)$	

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Section A (39 marks)

1. Simplify $\dfrac{\cos(\theta - 2\pi) + \cot\left(\dfrac{\pi}{2} - \theta\right)\sin\theta}{\sec(\pi + \theta)}$. (4 marks)

Answers written in the margins will not be marked.

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2. It is given that $A+B=\frac{\pi}{6}$.

(a) Prove that $\left(\sqrt{3}+\tan A\right)\left(\sqrt{3}+\tan B\right)=4$.

(b) Using the result of (a), find the value of $\tan\frac{\pi}{12}$.

(5 marks)

Answers written in the margins will not be marked.

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3. **(a)** Simplify $\frac{\sin 3x}{\sin x}-\frac{\cos 3x}{\cos x}$.

(b) Hence, find the value of $\frac{1}{\sin 10°}-\frac{\sqrt{3}}{\cos 10°}$ without using a calculator.

(5 marks)

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4. **(a)** Show that $\sin^6\theta+\cos^6\theta=\frac{1}{8}(5+3\cos 4\theta)$.

(b) Solve the equation $\sin^6\theta+\cos^6\theta=\frac{7}{16}$, where $0\le\theta\le 90°$.

(7 marks)

Answers written in the margins will not be marked.

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5. Evaluate $\sum_{n=6}^{10}\left(\sqrt{n+1}-\frac{n-2}{\sqrt{n+1}+\sqrt{3}}\right)$. (4 marks)

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6. Suppose the coefficients of x and x^2 in the expansion of $(1 + ax)^n$ are 35 and 525 respectively. Find the values of a and n. (5 marks)

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7. **(a)** Expand $243\left(\frac{x}{3}+2\right)^8$ in descending powers of x up to the term in x^6.

(b) The coefficient of x^7 in the expansion of $243(x+k)\left(\frac{x}{3}+2\right)^8$ is 32, where k is a constant. By using the result of **(a)**, find the value of k.

(4 marks)

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8. Prove, by mathematical induction, that

$$1^2 - 2^2 + 3^2 - 4^2 + \ldots + (2n+1)^2 = (2n+1)(n+1)$$

for all positive integers n. (5 marks)

Answers written in the margins will not be marked.

Section B (11 marks)

9. **(a)** Prove, by mathematical induction, that

$$1+3+6+\cdots+\frac{n(n+1)}{2}=\frac{n(n+1)(n+2)}{6}$$ for all positive integers n . (5 marks)

(b) Using the result of **(a)**, evaluate $\frac{19\times20}{2}+\frac{20\times21}{2}+\frac{21\times22}{2}+\cdots+\frac{45\times46}{2}$. (3 marks)

(c) Using the result of **(a)** and the fact that $1+2+3+\cdots+n=\frac{n(n+1)}{2}$, prove that

$$1^2+2^2+3^2+\cdots+n^2=\frac{n(n+1)(2n+1)}{6}$$ for all positive integers n . (3 marks)

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END OF PAPER

Answers written in the margins will not be marked.