Linear Inequalities in Two Unknowns

二元一次不等式

Exercises(練習)

1. Solve the following compound inequalities graphically.

在數線上表示下列各複合不等式的解。

(a) $x \geq 2$ and $x < 3$

(b) $x < -1$ and $x < 5$

(c) $x \leq 4$ and $x > -2$

(d) $x > 2$ and $x \leq -1$

2. Solve $5 < 3x + 2 \leq \frac{x}{2} + \frac{29}{2}$, and represent the solutions graphically.

解 $5 < 3x + 2 \leq \frac{x}{2} + \frac{29}{2}$，並在數線上表示其解。

3. Solve $2x - 5 < 9$ and $3x + 5 > 17$, and represent the solutions graphically.

解 $2x - 5 < 9$ 及 $3x + 5 > 17$，並在數線上表示其解。

4. Solve the system of inequalities $\begin{cases} 2x - y \leq 2 \\ x + 2y < 4 \end{cases}$ graphically.

利用圖解法解 $\begin{cases} 2x - y \leq 2 \\ x + 2y < 4 \end{cases}$。

5 Solve the inequality $x + 4y \geq 4$ graphically.

利用圖解法解不等式 $x + 4y \geq 4$。

6. Solve the inequality $x - 2y < 4$ graphically.

利用圖解法解不等式 $x - 2y < 4$。

7. Solve the system of inequalities $\begin{cases} x-2y\le 3 \\ x+3y<3 \\ x\ge -2 \end{cases}$ graphically.

利用圖解法解不等式組 $\begin{cases} x-2y\le 3 \\ x+3y<3 \\ x\ge -2 \end{cases}$。

8. Solve the system of inequalities $\begin{cases} x<y \\ x\le 2 \\ y<3 \end{cases}$ graphically.

利用圖解法解不等式組 $\begin{cases} x<y \\ x\le 2 \\ y<3 \end{cases}$。

9. (a) Draw and shade the region that satisfies the following constraints:

把滿足下列約束條件的區域塗上陰影。

$$\begin{cases} y\le 4 \\ 2x-y-2\le 0 \\ 7x+5y\ge -15 \end{cases}$$

(b) Find the maximum value of the function $P = 2x + y$ subject to the constraints in (a).

根據 (a) 中的約束條件，求函數 $P = 2x + y$ 的極大值。

10 . (a) Draw and shade the region that satisfies the following constraints:

把滿足下列約束條件的區域塗上陰影。

$$\begin{cases} 4x-3y+5\ge 0 \\ 8x-15y-17\le 0 \\ 2x+3y\le 11 \end{cases}$$

(b) Find the maximum and the minimum value of $P = 4x + 2y$ subject to the constraints in (a) if x and y are

(i) real numbers,

(ii) integers.

根據 (a) 中的約束條件，在下列情況中，求 $P = 4x + 2y$ 的極大值和極小值。

(i) x 和 y 都是實數。

(ii) x 和 y 都是整數。

11. A factory produces two models of sofa *A* and *B* each day subject to the following conditions:

(i) The maximum number of sofa produced by the factory each day is 35.

(ii) Each sofa *A* and each sofa *B* require 4 man-hours and 3 man-hours to make respectively. Only 120 man-hours are available each day.

(iii) Each sofa *A* and each sofa *B* require 2 units and 3 units of materials to make respectively. Only 90 units of materials are available each day.

It is given that the factory produces *x* sofa *A* and *y* sofa *B* each day.

(a) Write down all the constraints on *x* and *y*.

(b) Indicate the solutions that satisfy the constraints in (a) on a coordinate plane.

(c) If the factory makes a profit of $175 and $200 by producing each sofa *A* and sofa *B* respectively, find the values of *x* and *y* in order to make the maximum profit. What is the maximum profit?

某工廠根據下列條件生產兩款沙發 *A* 和 *B*。

(i) 該工廠每天最多生產 35 張沙發。

(ii) 製造每張沙發 *A* 需要 4 個工時，而製造每張沙發 *B* 則需要 3 個工時。每天只有 120 個工時可供使用。

(iii) 製造每張沙發 *A* 需要 2 單位材料，而製造每張沙發 *B* 則需要 3 單位材料。每天只有 90 單位材料可供使用。

已知該工廠每天製造 *x* 張沙發 *A* 和 *y* 張沙發 *B*。

(a) 寫出所有關於 *x* 和 *y* 的約束條件。

(b) 在一個直角坐標平面上，標示滿足在 (a) 中所得的約束條件的區域。

(c) 若該工廠每生產一張沙發 *A* 和一張沙發 *B* 可分別賺取 $175 和 $200 的利潤，為了賺取最大的利潤，求 *x* 和 *y* 的值。最大利潤是多少？

12. A patient needs at least 24 units of vitamin C and 20 units of vitamin E daily. The following table shows the number of units of vitamins C and E in each gram of healthy food *P* and *Q* respectively.

Healthy food	**Vitamin C**	**Vitamin E**
***P* (per g)**	4 units	2 units
***Q* (per g)**	1 unit	3 units

It is given that each gram of healthy food *P* and *Q* cost $8 and $10 respectively. If the patient takes *x* g of food *P* and *y* g of food *Q* in order to meet his daily vitamins needs stated above,

(a) write down all the constraints on *x* and *y*,

(b) draw and shade the region that satisfies the constraints in (a) on a coordinate plane,

(c) find the weights of food *P* and *Q* the patient takes daily so as to minimize the cost. Hence, find the minimum cost.

一名病人每天最少需要攝取 24 單位的維他命 C 和 20 單位的維他命 E。下表所示為每 g 健康食品 *P* 和 *Q* 中分別含有的維他命數量。

健康食品	**維他命 C**	**維他命 E**

P (/g)	4 單位	2 單位
Q (/g)	1 單位	3 單位

已知每 g 健康食品 P 和 Q 的成本分別為 $8 和 $10。為滿足上述維他命的攝取量，假設該病人每人都會吃 x g 健康食品 P 和 y g 健康食品 Q。

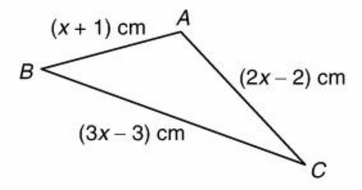
(a) 寫出所有關於 x 和 y 的約束條件。

(b) 在一個直角坐標平面上，繪出滿足在 (a) 中所得的約束條件的區域，並塗上陰影。

(c) 若要把成本減至最低，問該病人每天應分別吃多少健康食品 P 和 Q？由此，求最低成本。

13. The figure shows a triangle ABC, $AB = (x + 1)$ cm, $BC = (3x - 3)$ cm and $CA = (2x - 2)$ cm. It is given that the perimeter of $\triangle ABC$ is less than or equal to 38 cm but greater than 20 cm, find the range of the possible values of x.

圖中所示為 $\triangle ABC$，$AB = (x + 1)$ cm，$BC = (3x - 3)$ cm 及 $CA = (2x - 2)$ cm。已知 $\triangle ABC$ 的周界小於或等於 38 cm，但大於 20 cm，求 x 值的可能範圍。



14. A domestic electrical goods supplier has to transport 25 and 15 TV sets to shops A and B respectively from its two warehouses P and Q. The costs of transportation from the warehouses to the shops are shown in the following table:

From \ To	Shop A	Shop B
Warehouse P	$25/set	$15/set
Warehouse Q	$5/set	$10/set

It is given that there are 35 and 15 TV sets in stock in warehouses P and Q respectively. Suppose x and y TV sets are transported to shops A and B from warehouse P respectively.

(a) Write down all the constraints on x and y.

(b) Indicate the solutions that satisfy the constraints in (a) on a coordinate plane.

(c) Find the minimum total transportation cost.

某家庭電器供應商須分別運送 25 部和 15 部電視機到店鋪 A 和店鋪 B。已知該電器供應商有兩個倉庫 P 和 Q，而從各倉庫運送電視機到店鋪 A 和店鋪 B 的運費如下：

從 \ 到	店鋪 A	店鋪 B
倉庫 P	$25/部	$15/部
倉庫 Q	$5/部	$10/部

已知倉庫 P 和 Q 的電視機存貨量分別為 35 部和 15 部。假設從倉庫 P 分別運送了 x 部和 y 部電視機

往店鋪 A 和店鋪 B。

(a) 寫出所有關於 x 和 y 的約束條件。

(b) 在一個直角坐標平面上，標示滿足在 (a) 中所得的約束條件的區域。

(c) 求最低的總運費。

15. **(a)** Draw and shade the region that satisfies the following constraints:

把滿足下列約束條件的區域塗上陰影。

$$\begin{cases} x - y \geq 0 \\ x + y \leq 5 \\ 3x + y \geq 10 \end{cases}$$

(b) **(i)** Find the maximum value of the function $P = x + 2y + 3$ subject to the constraints in (a).

(ii) Does the function $P = x + 2y + 3$ have minimum value? Explain briefly.

(b) **(i)** 根據 (a) 中的約束條件，求函數 $P = x + 2y + 3$ 的極大值。

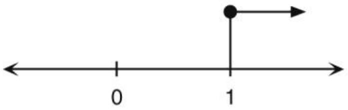
(ii) 問函數 $P = x + 2y + 3$ 有沒有極小值？試說明理由。

Pre-requisite Questions

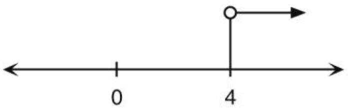
預備測驗

1. Set up an inequality in x for each of the following diagrams.
下列各圖分別代表一個不等式的解，試寫出該不等式。

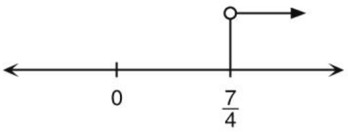
(a)



(b)



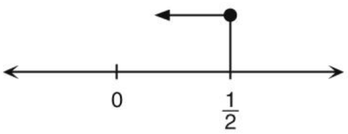
(c)



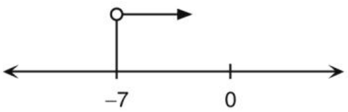
2 Solve $x + 3 > 4$, and represent the solution graphically.
解 $x + 3 > 4$，並在數線上表示其解。

3. Set up an inequality in x for each of the following diagrams.
下列各圖分別代表一個不等式的解，試寫出該不等式。

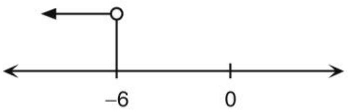
(a)



(b)



(c)



4. Solve $-3x + 4 < 7$, and represent the solution graphically.
解 $-3x + 4 < 7$，並在數線上表示其解。

5. Solve $x - 3 \leq 2$, and represent the solution graphically.
解 $x - 3 \leq 2$，並在數線上表示其解。

6. Solve $2x + 15 \geq 0$, and represent the solution graphically.
解 $2x + 15 \geq 0$，並在數線上表示其解。

7. Solve $6 - 2x < 1$, and represent the solution graphically.
解 $6 - 2x < 1$，並在數線上表示其解。

8. Solve $3x + 7 < \frac{5}{2}$, and represent the solution graphically.

解 $3x + 7 < \frac{5}{2}$，並在數線上表示其解。

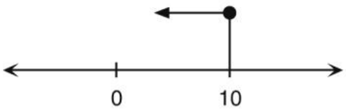
9. Solve $\frac{x}{5} + 1 \leq 0$, and represent the solution graphically.

解 $\frac{x}{5} + 1 \leq 0$，並在數線上表示其解。

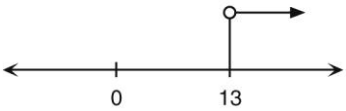
10. Solve $4x - 8 \geq -1$, and represent the solution graphically.
解 $4x - 8 \geq -1$，並在數線上表示其解。

11. Set up an inequality in x for each of the following diagrams.
下列各圖分別代表一個不等式的解，試寫出該不等式。

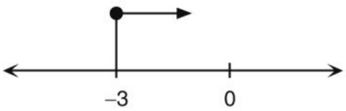
(a)



(b)



(c)



12. Solve $3x - 20 \geq -7$, and represent the solution graphically.
解 $3x - 20 \geq -7$，並在數線上表示其解。

13. Solve $7x + 12 \geq -2$, and represent the solution graphically.
解 $7x + 12 \geq -2$ ，並在數線上表示其解。

14. Solve $5x+18\le\frac{1}{2}$, and represent the solution graphically.

解 $5x+18\le\frac{1}{2}$，並在數線上表示其解。

15. Solve $-4x + 7 < 0$, and represent the solution graphically.

解 $-4x + 7 < 0$，並在數線上表示其解。

16. Solve $\frac{x}{8}+\frac{1}{2}<\frac{3}{8}$, and represent the solution graphically.

解 $\frac{x}{8}+\frac{1}{2}<\frac{3}{8}$，並在數線上表示其解。

17. Solve $4x - 2 \le -8$, and represent the solution graphically.

解 $4x - 2 \le -8$，並在數線上表示其解。

18. Solve $7 - 2x \ge 5$, and represent the solution graphically.

解 $7 - 2x \ge 5$，並在數線上表示其解。

19. Solve $-5x+\frac{11}{6}>-\frac{2}{3}$, and represent the solution graphically.

. 解 $-5x+\frac{11}{6}>-\frac{2}{3}$，並在數線上表示其解。

20. Solve $\frac{1}{12}(5x-9)\ge 6-\frac{x+3}{12}$, and represent the solution graphically.

解 $\frac{1}{12}(5x-9)\ge 6-\frac{x+3}{12}$，並在數線上表示其解。

21. Solve $\frac{x}{3}-\frac{1}{6}\ge\frac{7}{6}$, and represent the solution graphically.

解 $\frac{x}{3}-\frac{1}{6}\ge\frac{7}{6}$，並在數線上表示其解。

Level 1 Questions
程度 1 題目

1. Solve $2x + 3 < 1$ and $\frac{1}{2}-\frac{x}{4}<\frac{3}{2}$, and represent the solution graphically.

解 $2x + 3 < 1$ 及 $\frac{1}{2}-\frac{x}{4}<\frac{3}{2}$，並在數線上表示其解。

2. Solve $x - 3 \geq -1$ and $2x - 9 \leq 1$, and represent the solution graphically.

解 $x - 3 \geq -1$ 及 $2x - 9 \leq 1$，並在數線上表示其解。

3. Solve $\frac{x+5}{3}-1>0$ and $1 - x > -2$, and represent the solution graphically.

解 $\frac{x+5}{3}-1>0$ 及 $1 - x > -2$，並在數線上表示其解。

4. Solve $\frac{x}{3}+2>1$ and $-\frac{x}{6}+\frac{2}{3}\leq\frac{1}{2}$, and represent the solution graphically.

解 $\frac{x}{3}+2>1$ 及 $-\frac{x}{6}+\frac{2}{3}\leq\frac{1}{2}$，並在數線上表示其解。

5. Solve $-\frac{5}{6}\leq\frac{x}{3}+\frac{3}{2}\leq\frac{1}{6}$, and represent the solution graphically.

解 $-\frac{5}{6}\leq\frac{x}{3}+\frac{3}{2}\leq\frac{1}{6}$，並在數線上表示其解。

6. Solve $5 - x \leq 6$ and $\frac{x}{2}+\frac{1}{3}<\frac{11}{6}$, and represent the solution graphically.

解 $5 - x \leq 6$ 及 $\frac{x}{2}+\frac{1}{3}<\frac{11}{6}$，並在數線上表示其解。

7. Solve $-2x + 1 < 4$ and $\frac{x}{2}-1\leq 0$, and represent the solution graphically.

解 $-2x + 1 < 4$ 及 $\frac{x}{2}-1\leq 0$，並在數線上表示其解。

8. Solve $\begin{cases} 4-4x>1 \\ 1-\dfrac{x}{3}\le 2 \end{cases}$, and represent the solution graphically.

解 $\begin{cases} 4-4x>1 \\ 1-\dfrac{x}{3}\le 2 \end{cases}$，並在數線上表示其解。

9. Solve $-2 < 2x - 3 \le 2$, and represent the solution graphically.

解 $-2 < 2x - 3 \le 2$，並在數線上表示其解。

10. Solve $\begin{cases} \dfrac{x}{3}-1<1 \\ -\dfrac{x}{8}+\dfrac{1}{2}\le\dfrac{1}{4} \end{cases}$, and represent the solution graphically.

解 $\begin{cases} \dfrac{x}{3}-1<1 \\ -\dfrac{x}{8}+\dfrac{1}{2}\le\dfrac{1}{4} \end{cases}$，並在數線上表示其解。

11. Solve $\begin{cases} 1-2x>11 \\ \dfrac{x}{6}-\dfrac{1}{4}<\dfrac{1}{4} \end{cases}$, and represent the solution graphically.

解 $\begin{cases} 1-2x>11 \\ \dfrac{x}{6}-\dfrac{1}{4}<\dfrac{1}{4} \end{cases}$，並在數線上表示其解。

13. Solve $\begin{cases} 4x-5\ge 2 \\ \dfrac{1}{4}(3x+10)>1 \end{cases}$, and represent the solution graphically.

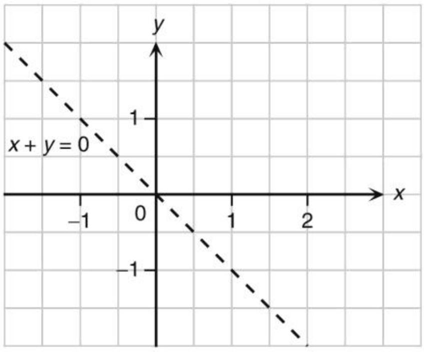
解 $\begin{cases} 4x-5\ge 2 \\ \dfrac{1}{4}(3x+10)>1 \end{cases}$，並在數線上表示其解。

14. Solve $\begin{cases} \dfrac{1}{2}-\dfrac{x}{6}\geq 0 \\ -\dfrac{x-8}{7}>1 \end{cases}$, and represent the solution graphically.

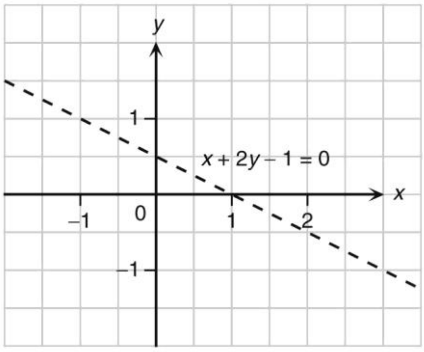
解 $\begin{cases} \dfrac{1}{2}-\dfrac{x}{6}\geq 0 \\ -\dfrac{x-8}{7}>1 \end{cases}$，並在數線上表示其解。

15. Shade the half-plane which satisfies the given inequality.
把滿足給定的不等式的半平面塗上陰影。

(a) $x + y > 0$

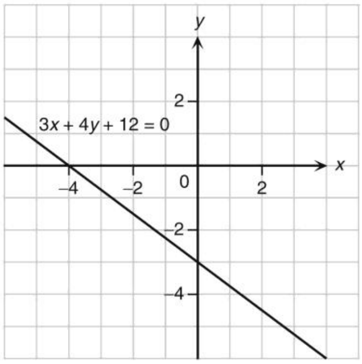


(b) $x + 2y < 1$

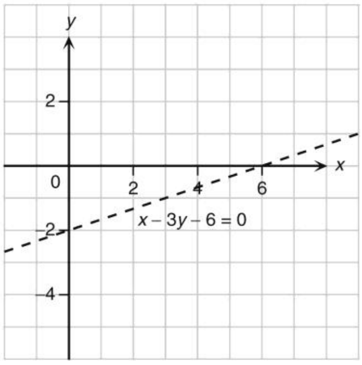


15. Shade the half-plane which satisfies the given inequality.
把滿足給定的不等式的半平面塗上陰影。

(a) $3x + 4y < 12$

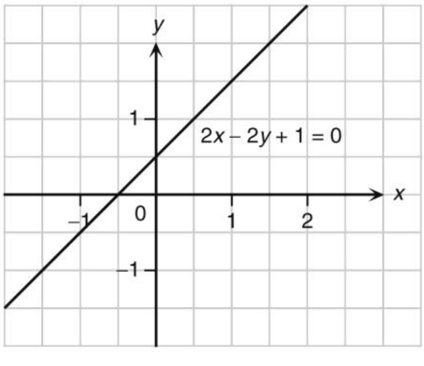


(b) $x - 3y < 6$

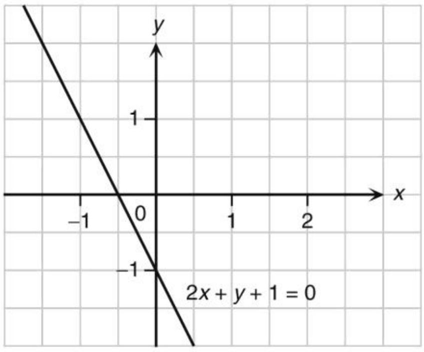


16. Shade the half-plane which satisfies the given inequality.
把滿足給定的不等式的半平面塗上陰影。

(a) $2x - 2y \le -1$

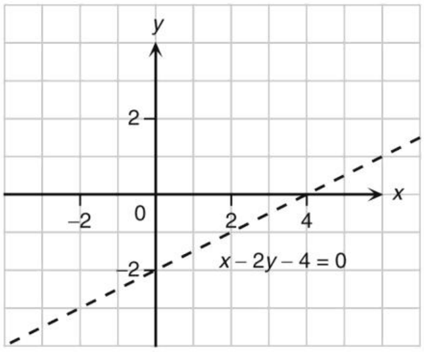


(b) $2x + y + 1 \le 0$

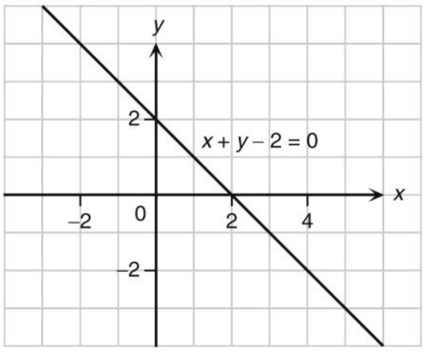


17. Shade the half-plane which satisfies the given inequality.
把滿足給定的不等式的半平面塗上陰影。

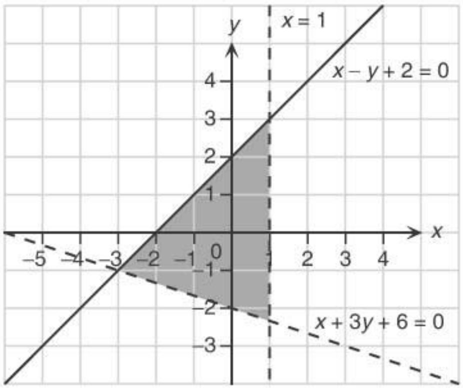
(a) $x - 2y - 4 < 0$



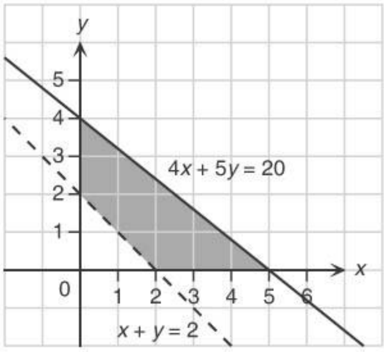
(b) $x + y - 2 \leq 0$



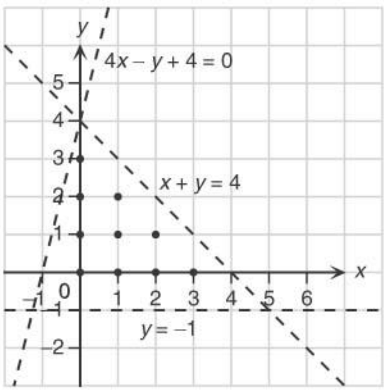
18. The shaded region shows the solutions of a system of inequalities. Write down the system of inequalities.
圖中的陰影區域所示為一個不等式組的解。寫出該不等式組。



19. The shaded region shows the solutions of a system of inequalities. Write down the system of inequalities.
圖中的陰影區域所示為一個不等式組的解。寫出該不等式組。

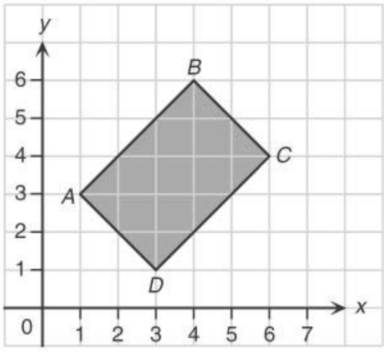


20. The dots in the figure show the solutions of a system of inequalities. Write down a possibility of the system of inequalities.
圖中的黑點所示為一個不等式組的解。寫出一個可能的不等式組。



21. The following region shows the solution of a system of inequalities. Write down the system of inequalities where the equations of AB, BC, CD and DA are given by:
下圖的陰影區域所示為一個不等式組的解，試寫出該不等式組。
其中AB、BC、CD 和 DA 的方程為:

$$\begin{cases} AB: x - y + 2 = 0 \\ BC: x + y - 10 = 0 \\ CD: x - y - 2 = 0 \\ DA: x + y - 4 = 0 \end{cases}$$



22. Solve the following system of inequalities graphically.
利用圖解法解下列的不等式組。

$$\begin{cases} x + 2y \geq -2 \\ 3x - 2y \geq 6 \end{cases}$$

23. Solve the following system of inequalities graphically.
利用圖解法解下列的不等式組。

$$\begin{cases} 2x + y + 2 > 0 \\ x - y - 1 < 0 \end{cases}$$

24. Solve the following system of inequalities graphically.
利用圖解法解下列的不等式組。

$$\begin{cases} x - 4y + 4 \leq 0 \\ 3x + 2y < 0 \end{cases}$$

25. Solve the following system of inequalities graphically.
利用圖解法解下列的不等式組。

$$\begin{cases} x + y \leq 1 \\ 2x - y \geq 3 \end{cases}$$

26. Solve the following system of inequalities graphically.
利用圖解法解下列的不等式組。

$$\begin{cases} x+3y>0 \\ 3x+2y\le 0 \end{cases}$$

27. Solve the following system of inequalities graphically.
利用圖解法解下列的不等式組。

$$\begin{cases} y\le 0 \\ x-y+2\ge 0 \\ 2x+y<2 \end{cases}$$

28. Solve the following system of inequalities graphically.
利用圖解法解下列的不等式組。

$$\begin{cases} x\le 0 \\ y\le 0 \\ 2x+3y+6>0 \end{cases}$$

29**.** Solve the following system of inequalities graphically.
利用圖解法解下列的不等式組。

$$\begin{cases} y\le 2 \\ x+y>2 \\ 2x-y<3 \end{cases}$$

30. Solve the following system of inequalities graphically.
利用圖解法解下列的不等式組。

$$\begin{cases} x\le 3 \\ y\ge x \\ 3x+2y\ge 6 \end{cases}$$

31. Solve the following system of inequalities graphically.
利用圖解法解下列的不等式組。

$$\begin{cases} y\ge -2 \\ 2x-y\ge -2 \\ 2x+y\le 4 \end{cases}$$

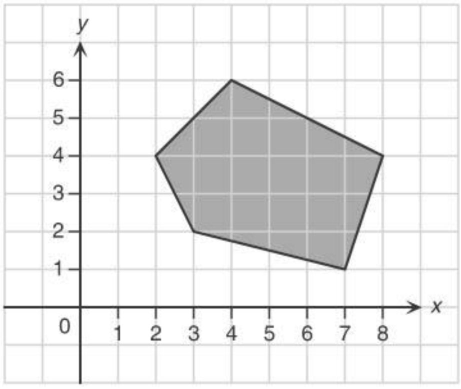
32. Solve the following system of inequalities graphically.

利用圖解法解下列的不等式組。

$$\begin{cases} 0 \le x < 2 \\ y \ge 0 \\ x + y \le 3 \end{cases}$$

33. The shaded region represents the solutions of a system of inequalities.

圖中陰影區域所示為一個不等式組的解。



(a) Find the maximum value of $P = 2x + 3y - 2$.

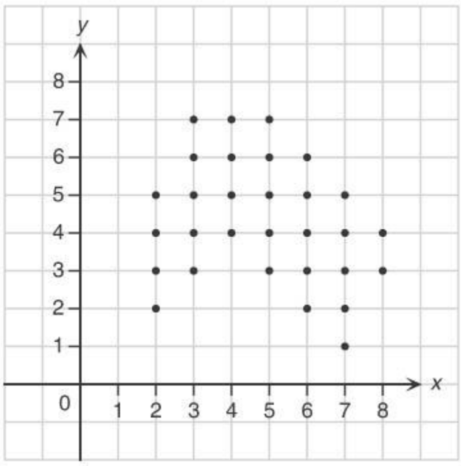
(b) Find the minimum value of $P = 2x + 3y - 2$.

(a) 求 $P = 2x + 3y - 2$ 的極大值。

(b) 求 $P = 2x + 3y - 2$ 的極小值。

34. The black dots in the figure represent the solutions of a system of inequalities.

圖中的黑點所示為一個不等式組的解。



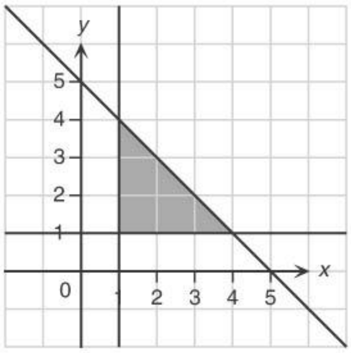
(a) Find the maximum value of $P = 2x - 4y$.

(b) Find the minimum value of $P = 2x - 4y + 3$.

(a) 求 $P = 2x - 4y$ 的極大值。

(b) 求 $P = 2x - 4y + 3$ 的極小值。

35. The shaded region represents the solutions of a system of inequalities.
圖中陰影區域所示為一個不等式組的解。



(a) Find the system of inequalities that has the solutions represented by the shaded region.

(b) Find the minimum value of $C = 2x + y + 1$.

(a) 求該不等式組。

(b) 求 $C = 2x + y + 1$ 的極小值。

36. The vitamin B content in 1 g of medicine P and 1 g of medicine Q are shown in the table.

	vitamin B_1	vitamin B_6	vitamin B_{12}
medicine P	20	30	30
medicine Q	30	20	10

Suppose a patient has to take vitamin B through medication. The minimum intake of vitamin B through medication for the patient is 120, 180 and 120 units of vitamin B_1, B_6 and B_{12} respectively. If the patient takes x g of medicine P and y g of medicine Q as the only source of vitamin B, write down all the constraints on x and y.

下表所示為 1 g 的藥物 P 和 1 g 的藥物 Q 所含的維他命 B。

	維他命 B_1	維他命 B_6	維他命 B_{12}
藥物 P	20	30	30
藥物 Q	30	20	10

假設一名病人的療程中須服用維他命 B，而他所需的維他命 B_1、B_6 和 B_{12} 的量分別是 120、180 和 120 單位。若病人的維他命 B 來源只有 P 和 Q 兩種藥物，寫出所有關於 x 和 y 的約束條件。

37. A craftsman makes tables and chairs, 10 hours a day and six days a week. There are 40 units of wood available. He needs 2 units of wood and 4 hours to make a table; 1 unit of wood and 3 hours to make a chair.

Suppose he makes x tables and y chairs in a week.

(a) Complete the following table.

	wood (units)	working hours
x tables		
y chairs		

(b) Write down all the constraints on x and y.

一名製造桌子和椅子的工匠，每週工作六天，每天工作 10 小時。現有 40 單位的木材。他需要 2 單位的木材和 4 小時來製造一張桌子。而製造一張椅子則需要 1 單位的木材和 3 小時。假設他在一週內製造 x 張桌子和 y 張椅子。

(a) 試完成下表。

	木材 (單位)	工時
x 張桌子		
y 張椅子		

(b) 寫出所有關於 x 和 y 的約束條件。

38. A baker makes two kinds of chocolate cakes, A and B. Each cake A is made of 5 eggs, 250 g of chocolate and 560 g of flour. Each cake B is made of 10 eggs, 200 g of chocolate and 800 g of flour. There are 60 eggs, 5 kg of chocolate and 10 kg of flour available. If the baker makes x cake A and y cake B, write down all the constraints on x and y.

一名麵包師傅製造 A 和 B 兩種巧克力蛋糕。每個蛋糕 A 由 5 隻雞蛋、250 g 的巧克力和 560 g 的麵粉製成而每個蛋糕 B 則由 10 隻雞蛋、200 g 的巧克力和 800 g 的麵粉製成。現有 60隻雞蛋、5 kg 的巧克力和 10 kg 的麵粉。若麵包師傅造了 x 個蛋糕 A 和 y 個蛋糕 B，寫出所有關於 x 和 y 的約束條件。

Level 2　Questions
程度 2　題目

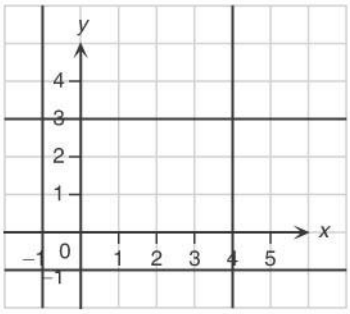
1. Solve the following system of inequalities graphically.
利用圖解法解下列的不等式組。

$$\begin{cases} x-3y\le 4 \\ 2x-y\ge -2 \\ 3x+4y\le 12 \end{cases}$$

2. Solve the following system of inequalities graphically.
利用圖解法解下列的不等式組。

$$\begin{cases} x-y\ge 0 \\ x-4y\le 0 \\ x+y\ge 4 \end{cases}$$

3. The figure shows the graphs of $y=3$, $y=-1$, $x=-1$ and $x=4$.
圖中所示為 $y=3$、$y=-1$、$x=-1$ 和 $x=4$ 的圖像。



(a) Add a straight line $x+y=3$ on the graph.

(b) Shade the region which represents the solutions of $\begin{cases} -1\le x\le 4 \\ -1\le y\le 3 \\ x+y\ge 3 \end{cases}$.

(c) If (x, y) is one of the solutions where x and y are positive integers, list all the ordered pairs (x, y).

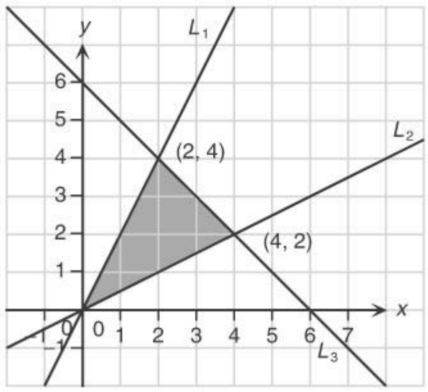
(a) 在圖中繪出直線 $x+y=3$。

(b) 把滿足不等式組 $\begin{cases} -1\le x\le 4 \\ -1\le y\le 3 \\ x+y\ge 3 \end{cases}$ 的區域塗上陰影。

(c) 若 (x, y) 是其中的一個解，其中 x 和 y 是正整數，列出所有序偶 (x, y)。

4. In the figure, (0, 0), (2, 4), (4, 2) are three vertices of the region bounded by the straight lines L_1, L_2 and L_3.

在圖中，(0, 0)、(2, 4)、(4, 2) 是由直線 L_1、L_2 和 L_3 所包圍的區域的三個頂點。



(a) Find the equations of L_1, L_2 and L_3.

(b) Write down a system of inequalities if the shaded region represents its feasible solutions.

(a) 求 L_1、L_2 和 L_3 的方程。

(b) 若陰影區域表示一個不等式組的可行解，寫出該不等式組。

5. It is given that $\begin{cases} x-2y-1\geq 0 \\ x+y\geq 3 \\ x\leq 4 \end{cases}$.

(a) Solve the system of inequalities graphically.

(b) List all the solutions (x, y) of the system of inequalities

(i) if x and y are integers.

(ii) if x and y are integers and $y < 1$.

已知 $\begin{cases} x-2y-1\geq 0 \\ x+y\geq 3 \\ x\leq 4 \end{cases}$。

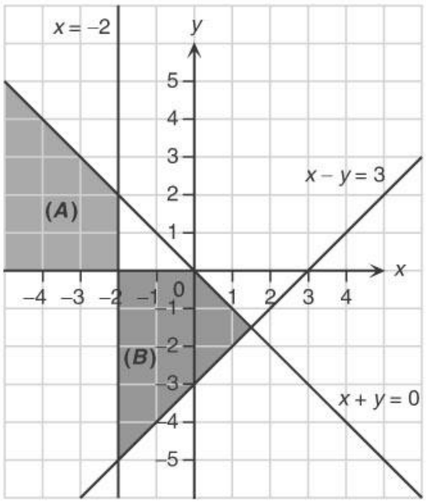
(a) 利用圖解法解該不等式組。

(b) 在下列情況中，列出該不等式組所有的解 (x, y)。

(i) x 和 y 都是整數。

(ii) x 和 y 都是整數及 $y < 1$。

6. Two shaded regions are shown in the figure.
圖中所示為兩個陰影區域。



(a) Write a system of inequalities with region (*A*) as its solutions.
(b) Write a system of inequalities with region (*B*) as its solutions.
(c) At the point of which region does the function $x - 2y$ attain its maximum value?
(a) 寫出一個不等式組，其中區域 (*A*) 是該不等式組的解。
(b) 寫出一個不等式組，其中區域 (*B*) 是該不等式組的解。
(c) 函數 $x - 2y$ 會在哪一個區域中的點達至其極大值？

7 **(a)** Draw and shade the region that satisfies the following constraints:
把滿足下列約束條件的區域塗上陰影。

$$\begin{cases} x \le 0 \\ x - 3y + 6 \ge 0 \\ x - y - 2 < 0 \end{cases}$$

(b) Find the maximum value of $P = x + y$ subject to the constraints in (a).
根據 (a) 中的約束條件，求 $P = x + y$ 的極大值。

8. **(a)** Draw and shade the region that satisfies the following constraints:
把滿足下列約束條件的區域塗上陰影。

$$\begin{cases} x \ge 0 \\ x - 2y \le 2 \\ 2x + 3y \le 6 \end{cases}$$

(b) **(b)** Find the maximum value of $P = x + 2y$ subject to the constraints in (a).
根據 (a) 中的約束條件，求 $P = x + 2y$ 的極大值。

9. **(a)** Draw and shade the region that satisfies the following constraints:
把滿足下列約束條件的區域塗上陰影。

$$\begin{cases} x \le 3 \\ y \ge 0 \\ 2x + 3y \le 9 \\ 3x - 2y \ge -2 \end{cases}$$

(b) Find the maximum value of $P = 2x + y$ subject to the constraints in (a).
根據 (a) 中的約束條件，求 $P = 2x + y$ 的極大值。

10. **(a)** Draw and shade the region that satisfies the following constraints:
把滿足下列約束條件的區域塗上陰影。

$$\begin{cases} x \ge 0 \\ x - 2y + 4 \ge 0 \\ 7x - 2y - 14 \le 0 \end{cases}$$

(b) Find the maximum value of $P = x + 3y$ subject to the constraints in (a).
根據 (a) 中的約束條件，求 $P = x + 3y$ 的極大值。

11. **(a)** Draw and shade the region that satisfies the following constraints:
把滿足下列約束條件的區域塗上陰影。

$$\begin{cases} 3x - 2y + 2 \ge 0 \\ 3x - 5y - 15 \le 0 \\ 3x + 5y - 15 \ge 0 \end{cases}$$

(b) Find the minimum value of $P = x + 2y$ subject to the constraints in (a).
根據 (a) 中的約束條件，求 $P = x + 2y$ 的極小值。

12. **(a)** Draw and shade the region that satisfies the following constraints:
把滿足下列約束條件的區域塗上陰影。

$$\begin{cases} x + y + 3 \ge 0 \\ x - 3y + 2 \ge 0 \\ 3x - 2y - 12 \le 0 \end{cases}$$

(b) Find the maximum and the minimum values of $P = 2x - 3y$ subject to the constraints in (a) if x and y are integers.
根據 (a) 中的約束條件，若 x 和 y 都是整數，求 $P = 2x - 3y$ 的極大值和極小值。

13. **(a)** Draw and shade the region that satisfies the following constraints:
把滿足下列約束條件的區域塗上陰影。

$$\begin{cases} x+3y-6\le 0 \\ 5x+2y-8\ge 0 \\ 2x-y-5\le 0 \end{cases}$$

(b) Find the maximum and the minimum values of $P = 3x + 2y$ subject to the constraints in (a).
根據 (a) 中的約束條件，求 $P = 3x + 2y$ 的極大值和極小值。

14. A triangular region ABC is bounded by three straight lines AB, BC and CA:

$AB: x-5y+13=0$
$BC: 4x+5y+2=0$
$CA: x=2$

(a) Find the three vertices of the triangular region.
(b) Write a system of linear inequalities with the triangular region as the region of feasible solutions.
(c) Hence, find the maximum value of $2x + 3y$ subject to the constraints found in (b).

一個三角區域 ABC 由三條直線 AB、BC 和 CA 所圍成：

$AB: x-5y-13=0$
$BC: 4x+5y+2=0$
$CA: x=2$

(a) 求三角區域的三個頂點。
(b) 若三角區域表示一個不等式組的可行解，寫出該不等式組。
(c) 由此，根據在 (b) 中的約束條件，求 $2x + 3y$ 的極大值。

15. In a professional examination, candidates should attempt two papers, paper A and paper B. There are 10 questions in each paper. The following table shows the marks and time required for each question in the two papers.

	marks	time (min)
paper A	9	16
paper B	15	20

Each candidate must attempt at least any 9 questions from the two papers in $2\frac{2}{3}$ hours. Under this condition, a candidate attempts x questions in paper A and y questions in paper B. Assume the candidate answers each question correctly.

(a) How many questions in each paper should he attempt in order to maximize his total score?

(b) What is the maximum total score?

在一個專業考試中，考生須回答 A 和 B 兩份考卷。每份考卷各有 10 題題目。下表所示為作答兩份考卷中各題的分數和所需的時間。

	分數	時間（分）
考卷 A	9	16
考卷 B	15	20

每位考生須在 $2\frac{2}{3}$ 小時內回答兩份考卷中任意最少 9 題題目。有一名考生按這限制在考卷 A 中回答了 x 題題目，在考卷 B 中則回答了 y 題題目。假設考生答對全部題目。

(a) 要取得最高分數，他須在每份考卷中分別回答多少題題目？

(b) 最高的總分是多少？

16. A fat man on a diet needs at least 20, 24 and 24 units of protein, carbohydrate and dietary fibre respectively per day. A unit of food P of price \$9 contains 10, 4 and 2 units of protein, carbohydrate and dietary fibre respectively. A unit of food Q of price \$6 contains 2, 4 and 8 units of protein, carbohydrate and dietary fibre respectively.

(a) Suppose he eats x units of P and y units of Q per day only, write down a system of inequalities on x and y.

(b) Solve the inequalities graphically and shade the region of feasible solutions.

(c) How much of each food does he need to eat each day to meet his dietary need at a minimum cost?

一名減肥中的胖子每天需要攝取的蛋白質、碳水化合物和食用纖維的最低量分別為 20、24 和 24 單位。每單位食品 P 售 \$9，所含的蛋白質、碳水化合物和食用纖維量分別為 10、4 和 2 單位。另一種食品 Q 每單位售 \$6，所含的蛋白質、碳水化合物和食用纖維量分別為 2、4 和 8 單位。

(a) 假設他每天只進食 x 單位的食品 P 和 y 單位的食品 Q，寫出一個關於 x 和 y 的不等式組。

(b) 利用圖解法解該不等式組，並把可行解區域塗上陰影。

(c) 他每天分別要進食多少食品 P 和 Q，才能以最低花費滿足每天所需的營養攝取量？

17. 10 g of alloy F contains 2 g of copper, 1 g of zinc and 1 g of lead; 10 g of alloy G contains 1 g of copper, 1 g of zinc and 3 g of lead. It is required to produce a new alloy which contains at least 10 g of copper, 8 g of zinc and 12 g of lead. Alloy G costs 1.5 times as much per gram as alloy F.
Find the amounts of alloy F and alloy G which must be mixed in order to produce the new alloy in the cheapest possible way.
10 g 的合金 F 含有 2 g 的銅、1 g 的鋅和 1 g 的鉛；10 g 的合金 G 則含有 1 g 的銅、1 g 的鋅和 3 g 的鉛。現需要生產一種新的合金，其中必須包含最少 10 g 的銅、8 g 的鋅和 12 g 的鉛。合金 G 的成本是合金 F 的 1.5 倍。
要以最低成本生產這種新的合金，需要多少合金 F 和合金 G？

18. A manufacturer employs 5 skilled workers and 10 semi-skilled workers to produce products in two qualities, the deluxe model and the ordinary model. It requires a skilled worker 2 hours and a semi-skilled worker 2 hours to make a deluxe product; and it requires a skilled worker 1 hour and a semi-skilled worker 3 hours to make an ordinary product. Every worker work 8 hours or less per day. Suppose the profit of a deluxe product is \$100 and that of an ordinary one is \$80. How many of each type should be produced each day in order to maximize the total daily profit?
一個生產商僱用了 5 名熟手技工，10 名半熟手技工來製造一批高價產品和一批低價產品。製造一件高價產品需要兩批技工各工作 2 小時。製造一件低價產品則需要熟手技工工作 1 小時和半熟手技工工作 3 小時。每名技工每天工作不多於 8 小時。設一件高價產品的利潤為 \$100，而一件低價產品的利潤則為 \$80。要獲得最大的利潤，這兩種產品每天的產量應各為多少？

19. A company manufactures two products.
Product P is made by mixing chemicals A, B and C in a ratio 4 : 1 : 2 by weight.
Product G is made by mixing chemicals A, B and C in a ratio 3 : 1 : 3 by weight.
The profit of making and selling 1 kg of product P is \$40 and that of product G is \$45. Suppose 240 kg of A, 70 kg of B and 180 kg of C are available, x kg of P and y kg of Q are made.

(a) Complete the following table and write down all the inequalities on x and y.

	product P (x kg)	product G (y kg)
content of chemical A (kg)		
content of chemical B (kg)		
content of chemical C (kg)		

(b) Find the maximum profit.

一間公司生產兩種產品。
產品 P 由三種化學品 A、B 和 C 按 4 : 1 : 2 重量比混合而成。
產品 G 由三種化學品 A、B 和 C 按 3 : 1 : 3 重量比混合而成。
生產和出售 1 kg 產品 P 和 G 的利潤分別為 \$40 和 \$45。現有 240 kg 的 A、70 kg 的 B 和 180 kg 的 C，可生產 x kg 的 P 和 y kg 的 Q。

(a) 完成下表，並寫出所有關於 x 和 y 的約束條件。

	產品 P (x kg)	產品 G (y kg)
化學品 A 的成分 (kg)		
化學品 B 的成分 (kg)		
化學品 C 的成分 (kg)		

(b) 求最大利潤。

20. A supermarket has two brands of sweets with wine, mint and nuts centres in stock. The composition of each brand of sweets is listed in the table below.

	wine centres	mint centres	nuts centres
a box of brand A (100 g)	25%	25%	50%
a box of brand B (100 g)	50%	25%	25%

The cost of brand A is \$60 per box and that of brand B is \$100 per box. The supermarket wants to combine x boxes of brand A and y boxes of brand B to make a new package containing at least 100 g of sweets with wine centres, 75 g with mint centres and 100 g with nuts centres.

(a) Write down all the constraints on x and y.

(b) What is the least cost of the new package of sweets?

一間超級市場出售兩種糖果盒，每種糖果盒皆有酒心、薄荷和果仁三種口味。下表所示為兩種糖果盒中的糖果分量。

	酒心	薄荷	果仁
A 牌糖果盒 (100 g)	25%	25%	50%
B 牌糖果盒 (100 g)	50%	25%	25%

每盒 A 牌糖果盒的成本為 \$60，而每盒 B 牌糖果盒的成本則為 \$100。超級市場打算把 x 盒 A 牌糖果盒和 y 盒 B 牌糖果盒混合，推出一種新包裝的糖果盒，每盒須有最少 100 g 酒心糖果、75 g 薄荷糖果和 100 g 果仁糖果。

(a) 寫出所有關於 x 和 y 的約束條件。

(b) 這種新包裝的糖果最低成本為多少？

Level 2+ Questions
程度 2+ 題目

1. A company plans to send Christmas cards to its customers. A box of card A costs \$25 and it contains 20 cards. A box of card B costs \$37.5 and it contains 40 cards. The company decides not to spend more than \$300 to buy at least 240 cards for its customers and plans to buy at least 3 boxes of each but not more than 10 boxes altogether.

(a) If the company buys x boxes of card A and y boxes of card B, what are the constraints on x and y?

(b) Solve the constraints in (a) graphically and hence list all the integral ordered pairs (x, y) that satisfy the constraints.

(c) What is the minimum cost for buying Christmas cards?

(d) Suppose free delivery is provided for a purchase of \$300 or above. The company then decides to spend exactly \$300 on the purchase of the cards.

(i) Find the number of boxes of card A card B the company should buy respectively.

(ii) Find the greatest number of cards the company can buy.

一間公司計劃寄出聖誕卡給客戶。一盒 20 張的聖誕卡 A 的售價為 \$25，而一盒 40 張的聖誕卡 B 的售價則為 \$37.5。公司須購買至少 240 張聖誕卡給客戶，成本預算最多為 \$300。公司計劃每款聖誕卡最少購買 3 盒，但合共不會買超過 10 盒。

(a) 若該公司買了 x 盒聖誕卡 A 和 y 盒聖誕卡 B，寫出所有關於 x 和 y 的約束條件。

(b) 利用圖解法解 (a) 中的約束條件，由此列出所有滿足這些約束條件的序偶 (x, y)。

(c) 購買聖誕卡的最低消費是多少？

(d) 設購物滿 \$300 或以上可獲免費送貨服務，於是公司決定花費剛好 \$300 來購買聖誕卡。

(i) 聖誕卡 A 和 B 各應購買多少盒？

(ii) 該公司最多可購買多少張聖誕卡？

2. In an activity day, 400 senior-form students have a picnic in Clear Water Bay while 500 junior-form students have activities in Victoria Park. A fast-food restaurant with branches A and B receives an order to supply lunch boxes to the students. Branch A can supply 700 lunch boxes and branch B can supply 400 lunch boxes. The service charges are shown in the table below.

	To Clear Water Bay	To Victoria Park
From Branch A	\$2 per box	\$1 per box
From Branch B	\$1 per box	\$0.5 per box

Suppose x lunch boxes are supplied from Branch A to Clear Water Bay, and

y lunch boxes are supplied from Branch A to Victoria Park.

(a) Write down all the constraints on x and y.

(b) Find the total service charge in terms of x and y.

(c) How should the delivery be made in order to obtain the most service charges?

活動日當天，400名高年級學生到清水灣郊遊，而 500名低年級學生則到維多少亞公園遊玩。一間快餐店接獲訂單，要為這些學生供應午飯飯盒。該快餐店共有兩間分店，A 和 B。A 店可以供應 700 個飯盒，而 B 店則可供應 400 個飯盒。下表所示為兩間店收取的服務費。

	到清水灣	到維多利亞公園
A 店送貨	每個飯盒 $2	每個飯盒 $1
B 店送貨	每個飯盒 $1	每個飯盒 $0.5

設 A 店供應 x 個飯盒到清水灣及 y 個飯盒到維多利亞公園。

(a) 寫出所有關於 x 和 y 的約束條件。

(b) 以 x 和 y 表示總服務費。

(c) 要獲得最多的服務費，快餐店應如何送貨？

3. A bakery bakes and sells two kinds of cakes, the chocolate cake and the cheesecake. The selling prices of one piece of chocolate cake and cheesecake are $5 and $8 respectively. A piece of chocolate cake is made of 3 parts chocolate, 2 parts sugar, 2 parts cream cheese, flour and water. A piece of cheesecake is made of 4 parts sugar, 5 parts cream cheese, flour and water. The cakes can be sold in fractions. Sarah buys chocolate cakes and cheesecakes as snack everyday. Sarah wants to eat at least 6 parts chocolate, 10 parts sugar and 8 parts cream cheese from the snack everyday.

(a) Let x and y be the numbers of pieces of chocolate cake and cheesecake Sarah eats every day. Write down the objective function to be optimized and the constraints on the objective function so that Sarah can minimize her expense on the cakes.

(b) Using graphical methods, find the quantity of chocolate cake and cheesecake to meet the Sarah's need at a minimum expense. Hence find the minimum expense.

The baker buys the raw materials including 600 parts chocolate, 1000 parts sugar and 800 parts cream cheese from a supplier. The supplier knows the contents of chocolate cake and cheesecake. He wants to sell the raw materials at the highest price the baker can afford. The highest cost of the raw materials the baker can afford is 20% of the selling price of the cakes.

(c) Let u_1, u_2 and u_3 be the prices of each part of chocolate, sugar and cream cheese respectively. Write down the objective function to be optimized and the constraints on the objective function so that the supplier can maximize its income by selling the raw materials.

一個麵包師傅製造及售賣巧克力蛋糕和芝士蛋糕。一件巧克力蛋糕售 $5，而一件芝士蛋糕售 $8。一件巧克力蛋糕由 3 份巧克力、2 份糖、2 份忌廉芝士、麵粉和水製成，而一件芝士蛋糕則由 4 份糖、5 份忌廉芝士、麵粉和水製成。每件蛋糕均可切開出售。美寶每天都會購買巧克力蛋糕和芝士蛋糕作小吃，並每天最少會吃 6 份巧克力、10 份糖和 8 份忌廉芝士。

(a) 設美寶每天吃 x 件巧克力蛋糕和 y 件芝士蛋糕。
寫出需優化的目標函數及相關的約束條件，讓美寶能夠把購買蛋糕的花費減到最低。

(b) 若美寶要儘量省錢，利用圖解法求她每天可吃多少件巧克力蛋糕和芝士蛋糕。由此求最低消費。

麵包師傅從供應商購入材料製造蛋糕，包括 600 份巧克力、1000 份糖和 800 份忌廉芝士。該供應商清楚知道製造巧克力蛋糕和芝士蛋糕的成分。他希望以麵包師傅能夠負擔的最高價錢售出材料。而麵包師傅能夠負擔的最高材料價錢是蛋糕售價的 20%。

(c) 設 u_1、u_2 和 u_3 為每份巧克力、糖和忌廉芝士的價錢。寫出需優化的目標函數及相關的約束條件，讓供應商能夠賺取最高收入。

4. A company has two warehouses, A and B, and two retail outlets, P and Q. The warehouses supply goods for the outlets for everyday sales.

A and B can store 12 units and 8 units of goods respectively. Both outlets P and Q sell 9 units of goods everyday.

The unit delivery cost of goods from A to P is twice that from A to Q, the unit delivery cost from B to P is 1.2 times that from B to Q, and the unit delivery cost from B to P is 1.5 times that from A to Q.

For every single delivery, the amount of goods delivered should be an integral number of units.

(a) Suppose the unit delivery cost from B to Q is c. Express the unit delivery costs from A to P and Q as well as from B to P in terms of c.

(b) Let x and y be the units of goods delivered from A to P and from A to Q respectively. Write down all the constraints on x and y.

(c) Hence devise a strategy for the delivery of goods so that the total delivery cost is a minimum.

(d) Suppose the unit delivery cost from B to P is k times that from B to Q, where $k \geq 1.2$, with other conditions remain unchanged. Show that the strategy in (c) remains unchanged for $k \leq 1.5$.

一間公司擁有 A 和 B 兩個倉庫，及 P 和 Q 兩所零售店。倉庫為零售店提供每天出售的貨品。

A 和 B 分別可存放 12 單位和 8 單位的貨品。零售店 P 和 Q 每天各售出 9 單位的貨品。

從 A 運送每單位貨品到 P 的運費是從 A 送貨到 Q 的兩倍，從 B 運送每單位貨品到 P 的運費是從 B 送貨到 Q 的 1.2 倍，而從 B 運送每單位貨品到 P 的運費是從 A 送貨到 Q 的 1.5 倍。每次送貨，送貨單位須為整數。

(a) 設從 B 運送每單位貨品到 Q 的運費是 c，試以 c 表示從 A 送貨到 P 和 Q 及從 B 送貨到 P 的運費。

(b) 設 x 和 y 分別為從 A 送到 P 和從 A 送到 Q 的貨品單位。寫出所有關於 x 和 y 的約束條件。

(c) 由此訂立出送貨編制，使總運費可以減到最低。

(d) 設從 B 運送每單位貨品到 P 的運費是從 B 送貨到 Q 的 k 倍，其中 $k \geq 1.2$，其他的條件則維持不變。證明若 $k \leq 1.5$，(c) 所訂立的送貨編制將維持不變。

5. An oil refinery refines both heavy crude oil and light crude oil into petroleum products. The table below shows the distribution of the natural yield of gasoline, kerosene and lubrication oil in heavy and light crude oil.

	gasoline	kerosene	lubrication oil
heavy crude	10%	17.5%	20%
light crude	20%	10%	16%

(Note: The percentages do not add to 100% because there are other products in the refining process not listed above.)

The costs of heavy and light crude oil per barrel are \$45 and \$38 respectively (1 barrel is 115.6 litres).
Suppose the refinery has to deliver 100 000 barrels of gasoline, 65 000 barrels of kerosene and 250 000 barrels of lubrication oil.
Let x and y be the numbers of units (100 000 barrels) of heavy and light crude oil bought for the refinery.

(a) Find the cost incurred in the refinery in terms of x and y.

(b) Write down all the constraints on x and y.

(c) Hence find the least total cost on crude oil that meets the production need.

一間煉油廠分別提煉重原油和輕原油成為石油製品。下表所示為重原油和輕原油能夠提煉的汽油、煤油和潤滑油分量。

	汽油	煤油	潤滑油
重原油	10%	17.5%	20%
輕原油	20%	10%	16%

（附註：因為尚有其他石油製品沒有列出，所以上表百分數的總和並非 100%。）
每桶重原油和輕原油的成本分別為 \$45 和 \$38。（1 桶即 115.6 升。）
設煉油廠需提煉 100 000 桶汽油、65 000 桶煤油和 250 000 桶潤滑油。
設 x 和 y 分別為重原油和輕原油購入的單位（每單位 100 000 桶）。

(a) 以 x 和 y 表示煉油廠的成本。

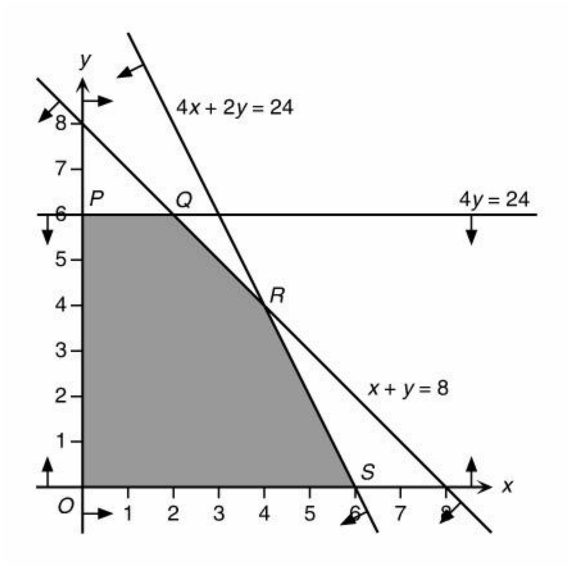
(b) 寫出所有關於 x 和 y 的約束條件。

(c) 由此，求達到預期產量的最低原油成本。

6. An objective function $z = cx + dy$ is maximized subject to the following constraints:
根據下列的約束條件，使一個目標函數 $z = cx + dy$ 達到其極大值。

$$\begin{cases} 4y \le 24 \\ x + y \le 8 \\ 4x + 2y \le 24 \\ x, y \ge 0 \end{cases}$$

The feasible region is shown in the following figure.
下圖所示為可行解。



At which points do the value of z attains its maximum if
在下列情況中，z 的值在哪點達到其極大值？

(a) both c and d are positive and
c 和 d 都是正數及

(i) $\frac{c}{d} > 2$?

(ii) $\frac{c}{d} = 2$?

(iii) $1 < \frac{c}{d} < 2$?

(iv) $\frac{c}{d} = 1$?

(v) $0 < \frac{c}{d} < 1$?

(b) **(i)** $c = 0, d > 0$?
(ii) $c > 0, d = 0$?
(iii) $c > 0, d < 0$?
(iv) $c < 0, d > 0$?
(v) $c < 0, d < 0$?

7. A furniture company produces two types of chairs, A and B. Every chair has to be assembled, finished and painted. The man-hours required for each process and the man-hours available for each process per month are given in the table.

	A	B	man-hours available per month
assembling	5	4	1900
finishing	1	1	500
painting	4	3	1500

The profit of making and selling each chair A is \$100, and that of chair B is \$60. Assume that all chairs produced can be sold.

(a) Let x and y be the numbers of chair A and B produced. Write down the objective function to be optimized and the constraints on the objective function so that the profit is a maximum.

(b) Hence, find the values of x and y so that the company can get the maximum possible profit.

一間家具公司生產 A 和 B 兩種椅子。每張椅子均需要裝嵌、拋光和油漆。下表所示為每個工序所需的工時及每月每個工序可用的工時。

	A	B	每月可用工時
裝嵌	5	4	1900
拋光	1	1	500
油漆	4	3	1500

製造並售出每張椅子 A 和椅子 B 的利潤分別是 \$100 和 \$60。假設所有製成的椅子皆可售出。

(a) 設 x 和 y 為椅子 A 和椅子 B 的生產數量。寫出需優化的目標函數及相關的約束條件，以獲得最大利潤。

(b) 由此，求 x 和 y 的值，讓該公司獲得最大利潤。

8. A paper company produces two types (type I and type II) of paper, type I has a width of 2 m, type II has a width of 2.5 m. It received an order of three models of paper with the following criteria:

Model	paper width (m)	paper length (m)
S	0.9	30 000
M	1.1	20 000
L	1.3	10 000

In order to satisfy the need of the customer, paper of different widths can be cut from type I and type II paper. For example, two rolls of paper of width 0.9 m may be cut from a roll of type I paper, with wastage of paper of width 0.2 m. Obviously, the paper company will not leave any wastage of paper of width 0.9 m or more.

(a) For the type I paper, there are three possible cutting methods. The first method cuts two rolls of paper of width 0.9 m, with wastage of paper of width 0.2 m. Complete the table to indicate the other two cutting methods.

	Cutting method 1	Cutting method 2	Cutting method 3
no. of rolls of *S*	2	1	0
no. of rolls of *M*	0		0
no. of rolls of *L*	0		
Wastage (m)	0.2	0	0.7

(b) For type II paper, there are five corresponding cutting methods. Complete the table.

	Cutting method 4	Cutting method 5	Cutting method 6	Cutting method 7	Cutting method 8
no. of rolls of *S*	2		0		
no. of rolls of *M*	0	2	1		
no. of rolls of *L*	0				
Wastage (m)	0.7	0.3		0.5	0.3

The paper company wants to minimize the area of wasted papers subject to the criteria set by the customer. Paper of widths 0.9 m, 1.1 m and 1.3 m produced that exceeds the demand of the customer is not considered as wastage.

(c) Let x_{11}, x_{12}, x_{13} be the lengths (m) of paper cut from type I paper according to cutting methods 1, 2 and 3 respectively. Let x_{24}, x_{25}, x_{26}, x_{27}, x_{28} be the lengths (m) of paper cut from type II paper according to cutting methods 4, 5, 6, 7 and 8 respectively. Write down the objective function to be optimized and the constraints on the objective function.

一間造紙公司生產兩種紙張，分別為 I 類和 II 類；I 類的闊度為 2 m，II 類的闊度則為 2.5 m。該 公司接獲一張生產三種尺寸的紙張的訂單，其要求如下：

紙張類別	紙張闊度 (m)	紙張長度 (m)
S	0.9	30 000
M	1.1	20 000
L	1.3	10 000

為符合顧客的要求，公司可從 I 類和 II 類紙張剪裁不同闊度的紙張。例如兩卷闊度為 0.9 m 的紙張可以從一卷 I 類紙張裁剪出來，但會浪費闊 0.2 m 的紙張。在這些條件下，公司不可能浪費闊 0.9 m 或以上的紙張。

(a) I 類紙張有三種裁剪方法。第一個方法是裁出兩卷闊度為 0.9 m 的紙張，損耗闊 0.2 m 的紙張。試完成下表以說明另外兩種的裁剪方法。

	裁剪方法 1	裁剪方法 2	裁剪方法 3
S 的卷數	2	1	0
M 的卷數	0		0
L 的卷數	0		
損耗 (m)	0.2	0	0.7

(b) II 類紙張則有五種裁剪方法。試完成下表。

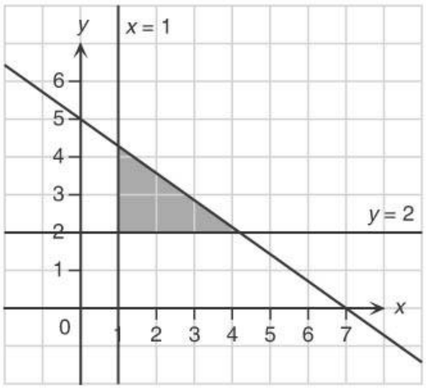
	裁剪方法 4	裁剪方法 5	裁剪方法 6	裁剪方法 7	裁剪方法 8
S 的卷數	2		0		
M 的卷數	0	2	1		
L 的卷數	0				
損耗 (m)	0.7	0.3		0.5	0.3

造紙公司要在符合顧客需求的情況下，把紙張損耗減至最低。另外，闊度為 0.9 m、1.1 m 和 1.3 m 的紙張的產量若多於客戶的需求，多餘的部分亦不當作損耗。

(c) 設 x_{11}、x_{12}、x_{13} 分別為利用裁剪方法 1、2 和 3 從 I 類紙張裁剪出來的紙張長度 (m)。設 x_{24}、x_{25}、x_{26}、x_{27} 和 x_{28} 分別為利用裁剪方法 4、5、6、7 和 8 從 II 類紙張裁剪出來的紙張長度 (m)。寫出需優化的目標函數及相關的約束條件。

Multiple Choice Questions
多項選擇題

1.



Which of the following systems of inequalities has the solutions that can be represented by the shaded region?
圖中的陰影區域代表下列哪一個不等式組的解？

A. $\begin{cases} x \geq 2 \\ y \geq 1 \\ 7x+5y \leq 35 \end{cases}$

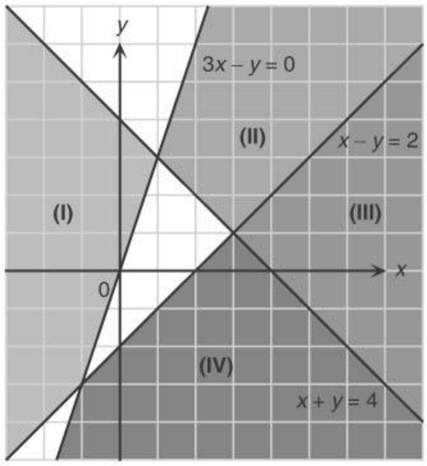
B. $\begin{cases} x \geq 1 \\ y \geq 2 \\ 5x+7y \leq 35 \end{cases}$

C. $\begin{cases} x \geq 1 \\ y \geq 2 \\ 7x+5y \leq 35 \end{cases}$

D. $\begin{cases} x \geq 2 \\ y \geq 1 \\ 5x+7y \geq 35 \end{cases}$

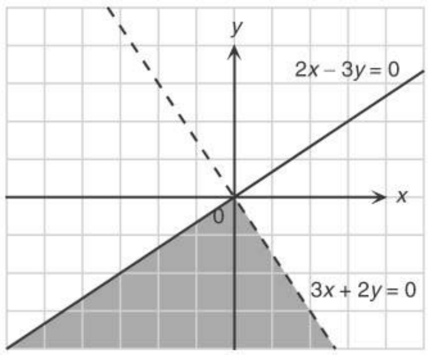
2. Which of the following regions represents the solutions of $\begin{cases} x+y \leq 4 \\ 3x-y \leq 0 \\ x-y \leq 2 \end{cases}$?

圖中的哪一個區域代表 $\begin{cases} x+y \leq 4 \\ 3x-y \leq 0 \\ x-y \leq 2 \end{cases}$ 的解？



A. Region I
B. Region II
C. Region III
D. Region IV

3.



Which of the following systems of inequalities has the solutions that can be represented by the shaded region?
圖中的陰影區域代表下列哪一個不等式組的解？

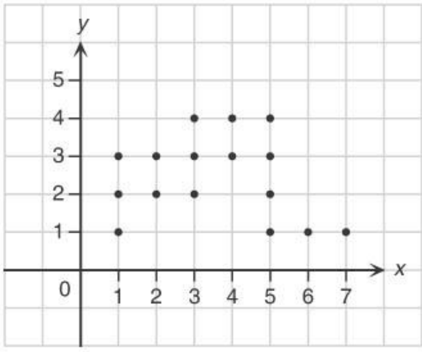
A. $\begin{cases} 2x-3y \geq 0 \\ 3x+2y > 0 \end{cases}$

B. $\begin{cases} 2x-3y \geq 0 \\ 3x+2y < 0 \end{cases}$

C. $\begin{cases} 2x-3y \leq 0 \\ 3x+2y > 0 \end{cases}$

D. $\begin{cases} 2x-3y \le 0 \\ 3x+2y < 0 \end{cases}$

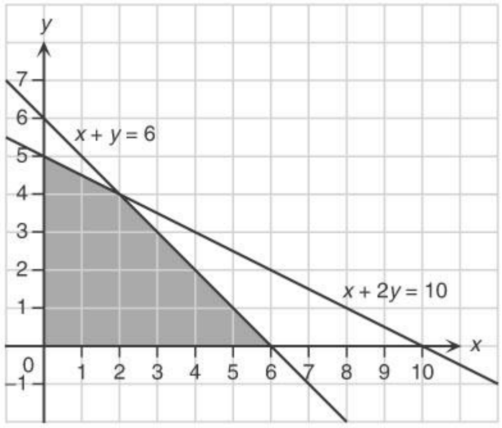
4. Among the points in the figure, at which point does $p = 3x + y$ attain its maximum value?
在圖中的各點之中，$p = 3x + y$ 在哪一點取得其極大值？



A. (7, 1)
B. (5, 4)
C. (3, 4)
D. (1, 3)

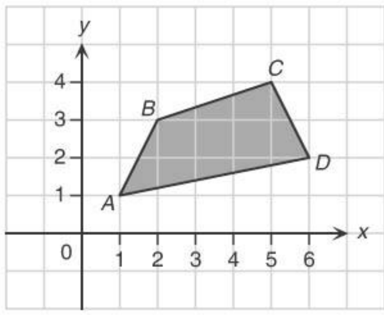
5. Find the maximum value of $p = 2x + y$ subject to the constraints $\begin{cases} x \ge 0 \\ y \ge 0 \\ x+2y \le 10 \\ x+y \le 6 \end{cases}$.

根據約束條件 $\begin{cases} x \ge 0 \\ y \ge 0 \\ x+2y \le 10 \\ x+y \le 6 \end{cases}$，求 $p = 2x + y$ 的極大值。



A. 6
B. 8
C. 12
D. 16

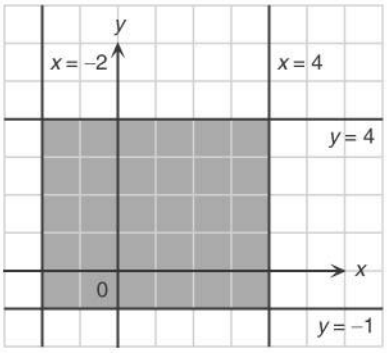
6. In the figure, at which point in the shaded region will the value of $2x + 3y - 5$ be the greatest?
問 $2x + 3y - 5$ 在圖中陰影區域內哪一點取得其極大值？



A. $A(1, 1)$
B. $B(2, 3)$
C. $C(5, 4)$
D. $D(6, 2)$

7. Find the minimum and maximum values of $x - 2y$ subject to the constraints $\begin{cases} -2 \le x \le 4 \\ -1 \le y \le 4 \end{cases}$.

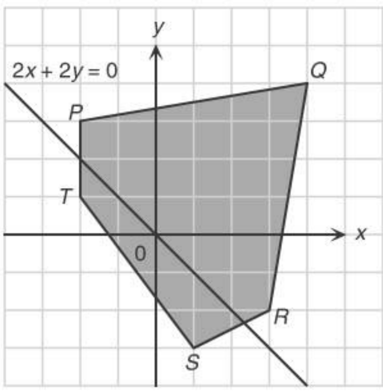
根據約束條件 $\begin{cases} -2 \le x \le 4 \\ -1 \le y \le 4 \end{cases}$，求 $x - 2y$ 的極小值和極大值。



A. –4, 1
B. –2, 4
C. –3, 8
D. –10, 6

8. In the figure, which point in the shaded region does $C = 2x + 2y$ attain its minimum value?

設 $C = 2x + 2y$。問 C 在圖中陰影區域內哪一點取得其極小值？



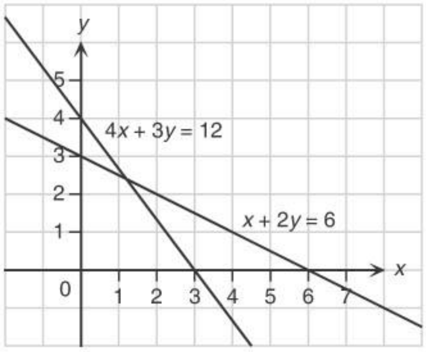
A. Point P
B. Point Q
C. Point R
D. Point S

9. In the figure, find the minimum value of $c = x + y$ subject to the following constraints:

$\begin{cases} 4x + 3y \ge 12 \\ x + 2y \ge 6 \\ x \text{ and } y \text{ are positive integers.} \end{cases}$

在圖中，根據下列的約束條件，求 $c = x + y$ 的極小值。

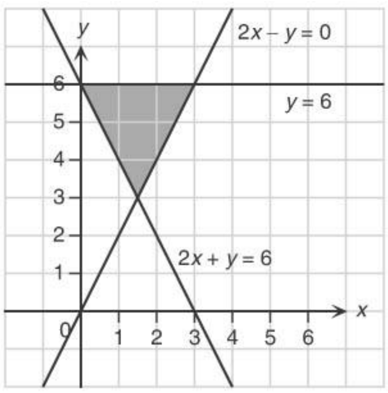
$\begin{cases} 4x + 3y \ge 12 \\ x + 2y \ge 6 \\ x \text{ 和 } y \text{ 都是正整數。} \end{cases}$



A. 3
B. 3.6
C. 4
D. 6

10. In the figure, if (x, y) is a point in the shaded region, which of the following constraints on x and y are true?

在圖中，若 (x, y) 是陰影區域內的一點，以下何者為正確？



I. $y \leq 6$

II. $2x - y \geq 0$

III. $2x + y \geq 6$

A. I and II only

B. I and III only

C. II and III only

D. I, II and III