Exponential and Logarithmic Functions

指數及對數函數

Exercises(練習)

1. Express each of the following in the form x^p, where p is a rational number.
把下列各題寫成 x^p 的形式，其中 p 是一個有理數。

(a) $\sqrt[7]{x^4}$

(b) $\dfrac{1}{(\sqrt[5]{x})^3}$

2. Simplify $\dfrac{(6x^3)^2}{(2x^2)^4}$ and express your answer with positive indices.

化簡 $\dfrac{(6x^3)^2}{(2x^2)^4}$，並以正指數表示答案。

3. Find the values of
求下列各題的值。

(a) $\sqrt[4]{16}$,

(b) $\sqrt[3]{-27}$.

4. Simplify $\left(\dfrac{x^3y^{-5}}{x^{-2}y^3}\right)^{-2}$ and express your answer with positive indices.

化簡 $\left(\dfrac{x^3y^{-5}}{x^{-2}y^3}\right)^{-2}$，並以正指數表示答案。

5. Find the values of
求下列各題的值。

(a) $(-125)^{\frac{1}{3}}$,

(b) $81^{-\frac{1}{2}}$.

6. Find the values of
求下列各題的值。

(a) $(-32)^{-\frac{3}{5}}$,

(b) $\left(\frac{64}{125}\right)^{-\frac{2}{3}}$.

7. Solve $5^{x-1}+5^{x}=30$.
解 $5^{x-1}+5^{x}=30$。

8. Simplify the following expressions and express your answers with positive indices.
化簡下列各題，並以正指數表示答案。

(a) $\sqrt[4]{x^{3}}\cdot\frac{x^{-2}}{\sqrt{x}}$

(b) $\frac{\sqrt[3]{x^{2}}}{\sqrt[4]{x^{3}}}\div(9x^{\frac{1}{2}})^{-\frac{1}{2}}$

9. Solve $16^{-x}=8^{\frac{5}{3}}$.

解 $16^{-x}=8^{\frac{5}{3}}$。

10. Solve the following simultaneous equations:
解以下的聯立方程：

$$\begin{cases}3^{3x-2y}=81\\2^{2x+3y}=128\end{cases}$$

11. Solve $2^{2x+1} - 3(2^x) + 1 = 0$.

解 $2^{2x+1} - 3(2^x) + 1 = 0$ 。

12. In each of the following, find the value of x correct to 3 significant figures.

求下列各題中 x 的值，準確至三位有效數字。

(a) $10^x = 4$

(b) $10^x = 7$

13. Find the values of the following common logarithms without using a calculator.

試不使用計算機，求下列各常用對數的值。

(a) log 10 000 000

(b) log 0.000 1

14. Find the values of the following logarithms without using a calculator.

試不使用計算機，求下列各對數的值。

(a) $\log_2 32$

(b) $\log_3 243$

15. Find the values of the following expressions without using a calculator.

試不使用計算機，求下列各對數的值。

(a) $\dfrac{\log 49}{\log \sqrt[3]{7}}$

(b) $\dfrac{\log_7 36}{\log_7 \sqrt{216}}$

16. Find the values of the following expressions without using a calculator.

試不使用計算機，求下列各對數的值。

(a) $\log 25 + \log 40$

(b) $\log_3 189 - \log_3 7$

17. Simplify $\dfrac{5\log x^3}{6\log x - \log \sqrt[3]{x}}$, where $x > 0$ and $x \neq 1$.

化簡 $\dfrac{5\log x^3}{6\log x - \log \sqrt[3]{x}}$ ，其中 $x > 0$ 且 $x \neq 1$。

18. Solve $3^{2x-1} = 16$ and give your answer correct to 3 significant figures.

解 $3^{2x-1} = 16$ ，並取答案準確至三位有效數字。

19. Given that log 2 = *a* and log 3 = *b*, express the following in terms of *a* and *b*.
已知 log 2 = *a* 及 log 3 = *b*，試以 *a* 和 *b* 表示下列各數式。

(a) log 6

(b) log 1800

20. Solve $(\log x)^2 - \log x - 6 = 0$.
解 $(\log x)^2 - \log x - 6 = 0$。

21. Solve $5^{x-1} = 3^{x+1}$ and give your answer correct to 3 significant figures.
解 $5^{x-1} = 3^{x+1}$，並取答案準確至三位有效數字。

22. Solve the following logarithmic equations.
解下列各對數方程。

(a) $\log(3x - 2) = 2$

(b) $\log(x-6) - \log(x-8) = \log 2$

23. Given that the intensity level of the sound produced by a barking dog is 50 dB and that of a crying baby is 45 dB. How many times is the intensity of the sound produced by the barking dog to that of the crying baby? (Give your answer correct to 3 significant figures.)
已知一隻狗的吠聲和一名嬰兒的哭聲的聲強級分別是 50 dB 和 45 dB。問狗的吠聲的聲強相當於嬰兒的哭聲的多少倍？(答案須準確至三位有效數字。)

24. The strengths of earthquakes in City *A* and City *B* were measured 6 and 5.4 respectively on the Richter scale. How many times is the strength of the earthquake in City *A* to that in City *B*? (Give your answer correct to 3 significant figures.)
已知發生於 *A* 城和 *B* 城的地震的強度分別為黎克特制 6 級和 5.4 級。問 *A* 城的地震的強度是

25. The sound intensity inside a concert hall is 3 times as that outside the concert hall. If the sound intensity level outside the concert hall is 45 dB, find the sound intensity level inside the concert hall. (Give your answer correct to 3 significant figures.)
已知演奏廳內的聲強是演奏廳外的 3 倍。若演奏廳外的聲強級是 45 dB，求演奏廳內的聲強級。(答案須準確至三位有效數字。)

26. Solve $4^{2x} \cdot 64^{x+1} = \left(\frac{1}{8}\right)^{-3x}$.
解 $4^{2x} \cdot 64^{x+1} = \left(\frac{1}{8}\right)^{-3x}$。

27. Simplify the following expression and express your answer with positive indices.
化簡下列各題，並以正指數表示答案。

$$\left(\frac{1}{9}\right)^{-\frac{1}{2}} \cdot \left(\frac{27a}{\sqrt{b^3}}\right)^{\frac{1}{3}} \cdot \sqrt[3]{\frac{8b^3}{a^7}}$$

28. Solve $6\log_2 (x+1)^{\frac{1}{3}} - \log_2 2x = 1.$

解 $6\log_2 (x+1)^{\frac{1}{3}} - \log_2 2x = 1$ 。

29. Simplify $\dfrac{3\log_2\left(\frac{3}{2}\right)^2 - \log_2 \frac{8}{27}}{\log_2 3 - \log_2 2}.$

化簡 $\dfrac{3\log_2\left(\frac{3}{2}\right)^2 - \log_2 \frac{8}{27}}{\log_2 3 - \log_2 2}$ 。

Pre-requisite Questions

預備測驗

1. Find the value of $5^3 \times 5^4 \div 5^5$.
求 $5^3 \times 5^4 \div 5^5$ 的值。

2. Find the value of $k^{-9} \times k^9$.
求 $k^{-9} \times k^9$ 的值。

3. Find the value of $(3^2)^3$.
求 $(3^2)^3$ 的值。

4. Find the value of $2^3 \times 2^2$.
求 $2^3 \times 2^2$ 的值。

5. Find the value of $4^6 \times 4^{-3}$.
求 $4^6 \times 4^{-3}$ 的值。

6. Find the value of $\left(-\frac{1}{7}\right)^0 \div 2^{-4}$.
求 $\left(-\frac{1}{7}\right)^0 \div 2^{-4}$ 的值。

7. Find the value of $\frac{5^0 - 4^{-1}}{2^{-2}}$.
求 $\frac{5^0 - 4^{-1}}{2^{-2}}$ 的值。

8. Find the value of $\left(\frac{6}{7}\right)^{-2} \times \left(\frac{7}{8}\right)^0 \times \left(\frac{4}{7}\right)^2$.
求 $\left(\frac{6}{7}\right)^{-2} \times \left(\frac{7}{8}\right)^0 \times \left(\frac{4}{7}\right)^2$ 的值。

9. Find the value of $(3^{-2} \times 3^0)^{-1}$.
求 $(3^{-2} \times 3^0)^{-1}$ 的值。

10. Find the value of $\left(-\frac{3}{2^{-1}}\right)^{-3}$.

求 $\left(-\frac{3}{2^{-1}}\right)^{-3}$ 的值。

11. Simplify and express $a^7 \div a^2 \times a^5$.

化簡 $a^7 \div a^2 \times a^5$。

12. Simplify and express $(-4a^3)^2$ with positive indices.

化簡 $(-4a^3)^2$，並以正指數表示答案。

13. Simplify and express $b^4 \div b^{-8}$ with positive indices.

化簡 $b^4 \div b^{-8}$，並以正指數表示答案。

14. Simplify and express $(3m^2n)^0(2m^{-2}n)^5$ with positive indices.

化簡 $(3m^2n)^0(2m^{-2}n)^5$，並以正指數表示答案。

15. Simplify and express $\left(\frac{1}{b}\right)^{-6}$ with positive indices.

化簡 $\left(\frac{1}{b}\right)^{-6}$，並以正指數表示答案。

16. Simplify and express $(a^2b^{-1})^{-3}$ with positive indices.

化簡 $(a^2b^{-1})^{-3}$，並以正指數表示答案。

17. Simplify and express $\frac{x}{y^{-1}}+\left(\frac{x^{-1}}{y}\right)^{-1}$ with positive indices.

化簡 $\frac{x}{y^{-1}}+\left(\frac{x^{-1}}{y}\right)^{-1}$，並以正指數表示答案。

18. Simplify and express $\left(\frac{r^0s^{-2}}{t^3}\right)^{-4}$ with positive indices.

化簡 $\left(\frac{r^0s^{-2}}{t^3}\right)^{-4}$，並以正指數表示答案。

Level 1 Questions
程度 1 題目

1. Simplify and express $\dfrac{(3a^3)^4}{(9a^2)^2}$ with positive indices.

化簡 $\dfrac{(3a^3)^4}{(9a^2)^2}$，並以正指數表示答案。

2. Simplify and express $\left(\dfrac{18^{-1}a^{-5}b^6}{6^{-2}a^5}\right)^{-2}$ with positive indices.

化簡 $\left(\dfrac{18^{-1}a^{-5}b^6}{6^{-2}a^5}\right)^{-2}$，並以正指數表示答案。

3. Simplify and express $\dfrac{(2b^{-1})^6}{(4^{-1}b^4)^{-4}}$ with positive indices.

化簡 $\dfrac{(2b^{-1})^6}{(4^{-1}b^4)^{-4}}$，並以正指數表示答案。

4. Simplify and express $\left(\dfrac{a^2b^{-2}}{a^{-5}b^3}\right)^3$ with positive indices.

化簡 $\left(\dfrac{a^2b^{-2}}{a^{-5}b^3}\right)^3$，並以正指數表示答案。

5. Find the value of $\sqrt[5]{-\left(\dfrac{1}{243}\right)}$.

求 $\sqrt[5]{-\left(\dfrac{1}{243}\right)}$ 的值。

6. Find the value of $\sqrt[6]{64}$.

求 $\sqrt[6]{64}$ 的值。

7. Find the value of $729^{-\frac{1}{3}}$.

求 $729^{-\frac{1}{3}}$ 的值。

8. Find the value of $256^{\frac{1}{4}}$.

求 $256^{\frac{1}{4}}$ 的值。

9. Express $\sqrt[5]{x^2}$ in the form x^p, where p is a rational number.

把 $\sqrt[5]{x^2}$ 寫成 x^p 的形式，其中 p 是一個有理數。

10. Express $(\sqrt[3]{x})^6$ in the form x^p, where p is a rational number.

把 $(\sqrt[3]{x})^6$ 寫成 x^p 的形式，其中 p 是一個有理數。

11. Express $\dfrac{9x}{\sqrt[4]{81x^6}}$ in the form x^p, where p is a rational number.

把 $\dfrac{9x}{\sqrt[4]{81x^6}}$ 寫成 x^p 的形式，其中 p 是一個有理數。

12. Express $\dfrac{2}{(\sqrt{x})^3}$ in the form x^p, where p is a rational number.

把 $\dfrac{2}{(\sqrt{x})^3}$ 寫成 x^p 的形式，其中 p 是一個有理數。

13. Find the value of $(-216)^{\frac{2}{3}}$ without using a calculator.

試不使用計算機，求 $(-216)^{\frac{2}{3}}$ 的值。

14. Find the value of $\left(-\dfrac{64}{125}\right)^{\frac{4}{3}}$ without using a calculator.

試不使用計算機，求 $\left(-\dfrac{64}{125}\right)^{\frac{4}{3}}$ 的值。

15. Find the value of $\left(\dfrac{625}{16}\right)^{\frac{3}{4}}$ without using a calculator.

試不使用計算機，求 $\left(\dfrac{625}{16}\right)^{\frac{3}{4}}$ 的值。

16. Simplify and express $\dfrac{\sqrt[4]{a^3}}{\sqrt[3]{a^2}}$ with positive indices.

化簡 $\dfrac{\sqrt[4]{a^3}}{\sqrt[3]{a^2}}$，並以正指數表示答案。

17. Find the value of $\left(\dfrac{36}{49}\right)^{-\frac{3}{2}}$ without using a calculator.

試不使用計算機，求 $\left(\dfrac{36}{49}\right)^{-\frac{3}{2}}$ 的值。

18. Simplify and express $(8a^{-3}b^4)^{-\frac{2}{3}} \div (a^{\frac{1}{2}}b^{-\frac{5}{6}})^4$ with positive indices.

化簡 $(8a^{-3}b^4)^{-\frac{2}{3}} \div (a^{\frac{1}{2}}b^{-\frac{5}{6}})^4$，並以正指數表示答案。

19. Simplify and express $\dfrac{\sqrt{144a^4}}{(2a^{-1})^3}$ with positive indices.

化簡 $\dfrac{\sqrt{144a^4}}{(2a^{-1})^3}$，並以正指數表示答案。

20. Simplify and express $(ab^{-2})^5 \cdot (a^6b^9)^{\frac{1}{3}}$ with positive indices.

化簡 $(ab^{-2})^5 \cdot (a^6b^9)^{\frac{1}{3}}$，並以正指數表示答案。

21. Simplify and express $(3a^{-\frac{1}{3}}b^{-\frac{4}{5}})^2 \div (-32a^{\frac{5}{6}}b^{\frac{1}{4}})^{-\frac{2}{5}}$ with positive indices.

化簡 $(3a^{-\frac{1}{3}}b^{-\frac{4}{5}})^2 \div (-32a^{\frac{5}{6}}b^{\frac{1}{4}})^{-\frac{2}{5}}$，並以正指數表示答案。

***22*.** Simplify and express $\left(\frac{81a^{-2}b^{\frac{2}{3}}}{256b^{-2}}\right)^{-\frac{3}{4}}$ with positive indices.

化簡 $\left(\frac{81a^{-2}b^{\frac{2}{3}}}{256b^{-2}}\right)^{-\frac{3}{4}}$，並以正指數表示答案。

***23*.** Solve the equation $4^{\frac{2x}{3}-1}=256^{\frac{3}{4}}$.

解 $4^{\frac{2x}{3}-1}=256^{\frac{3}{4}}$。

24 Solve the equation $3^{\frac{x}{2}}=243^{\frac{2}{5}}$.

. 解 $3^{\frac{x}{2}}=243^{\frac{2}{5}}$。

***25*.** Solve the equation $3^{x+3}-3(3^x)=72$.

解 $3^{x+3}-3(3^x)=72$。

***26*.** Solve the equation $9^{\frac{x}{4}}=27^{\frac{2}{3}}$.

解 $9^{\frac{x}{4}}=27^{\frac{2}{3}}$。

27. Solve the equation $4(4^{x-1})-\frac{4^x}{16}=240.$

解 $4(4^{x-1})-\frac{4^x}{16}=240$。

28. Solve the equation $9^{3x}=81(3^x)$.

解 $9^{3x}=81(3^x)$。

***29*.** Solve the equation $2^{2x+4}-4^{x-1}=126.$

解 $2^{2x+4}-4^{x-1}=126$。

***30*.** Solve the equation $3^{5x} = 27(9^x)$.
解 $3^{5x} = 27(9^x)$。

31. Find the value of log 1000 without using a calculator.
試不使用計算機，求 log 1000 的值。

***32*.** Find the value of $\log\frac{1}{100000}$ without using a calculator.
試不使用計算機，求 $\log\frac{1}{100000}$ 的值。

***33*.** Find the value of log 0.000 1 without using a calculator.
試不使用計算機，求 log 0.000 1 的值。

***34*.** Find the value of $\log_3\frac{1}{\sqrt{729}}$ without using a calculator.
試不使用計算機，求 $\log_3\frac{1}{\sqrt{729}}$ 的值。

35. Find the value of $\log_2 128$ without using a calculator.
試不使用計算機，求 $\log_2 128$ 的值。

***36*.** Find the value of $\log_5 0.008$ without using a calculator.
. 試不使用計算機，求 $\log_5 0.008$ 的值。

***37*.** Find the value of *x* in the equation $10^x = 7$ and give your answer correct to 3 significant figures.
解 $10^x = 7$，並取答案準確至三位有效數字。

38. Find the value of *x* in the equation $10^x = 40$ and give your answer correct to 3 significant figures.
解 $10^x = 40$，並取答案準確至三位有效數字。

***39*.** Find the value of *x* in the equation $10^{3x} = 80$ and give your answer correct to 3 significant figures.
解 $10^{3x} = 80$，並取答案準確至三位有效數字。

***40*.** Find the value of *x* in the equation $10^x = 169$ and give your answer correct to 3 significant figures.
解 $10^x = 169$，並取答案準確至三位有效數字。

***41*.** Find the value of x in the equation $10^x = 0.5$ and give your answer correct to 3 significant figures.
解 $10^x = 0.5$，並取答案準確至三位有效數字。

***42*.** Find the value of $\log 25 + \log 400$ without using a calculator.
試不使用計算機，求 $\log 25 + \log 400$ 的值。

***43*.** Find the value of x in the equation $(\sqrt{10})^x = 9.2$ and give your answer correct to 3 significant figures.
解 $(\sqrt{10})^x = 9.2$，並取答案準確至三位有效數字。

44. Find the value of $\log 8 + \log 625 - \log 5$ without using a calculator.
試不使用計算機，求 $\log 8 + \log 625 - \log 5$ 的值。

45. Find the value of $\log_4 \sqrt{2} - \log_4 \sqrt{32}$ without using a calculator.
試不使用計算機，求 $\log_4 \sqrt{2} - \log_4 \sqrt{32}$ 的值。

***46*.** Find the value of $\dfrac{\log_6 \sqrt[3]{16}}{\log_6\left(\dfrac{1}{64}\right)}$ without using a calculator.

試不使用計算機，求 $\dfrac{\log_6 \sqrt[3]{16}}{\log_6\left(\dfrac{1}{64}\right)}$ 的值。

47. Find the value of $\dfrac{\log_5 125}{\log_5 25}$ without using a calculator.

試不使用計算機，求 $\dfrac{\log_5 125}{\log_5 25}$ 的值。

***48*.** Find the value of $\dfrac{\log 81}{\log 243}$ without using a calculator.

試不使用計算機，求 $\dfrac{\log 81}{\log 243}$ 的值。

49. Given that $\log 3 = x$ and $\log 4 = y$, express $\log 12$ in terms of x and y.
已知 $\log 3 = x$ 及 $\log 4 = y$，試以 x 和 y 表示 $\log 12$。

50. Given that $\log 3 = x$ and $\log 4 = y$, express $\log 432$ in terms of x and y.
已知 $\log 3 = x$ 及 $\log 4 = y$，試以 x 和 y 表示 $\log 432$。

***51*.** Given that log 2 = x and log 3 = y, express log 135 in terms of x and y.
已知 log 2 = x 及 log 3 = y，試以 x 和 y 表示 log 135。

***52*.** Solve the equation $3^{x+1} = 8$ and give your answer correct to 3 significant figures.
解 $3^{x+1} = 8$，並取答案準確至三位有效數字。

53 Solve the equation $2(2^{x-1}) + 3(2^{x-2}) = 42$ and give your answer correct to 3 significant figures.
解 $2(2^{x-1}) + 3(2^{x-2}) = 42$。並取答案準確至三位有效數字。

***54*.** Given that log 2 = x and log 3 = y, express log 240 in terms of x and y.
已知 log 2 = x 及 log 3 = y，試以 x 和 y 表示 log 240。

55 Solve the equation $5^{3x} = 4^{2x-1}$ and give your answer correct to 3 significant figures.
解 $5^{3x} = 4^{2x-1}$，並取答案準確至三位有效數字。

***56*.** Solve the equation $2^{x-1}5^{2x} = 4$ and give your answer correct to 3 significant figures.
解 $2^{x-1}5^{2x} = 4$，並取答案準確至三位有效數字。

***57*.** Solve the equation $\log_4(5x-1) = -1$.
解 $\log_4(5x-1) = -1$。

58. Solve the equation log $(3x - 2) = 2$.
解 log $(3x - 2) = 2$。

***59*.** Solve the equation $\log_2(x+1) = 5$.
解 $\log_2(x+1) = 5$。

60. Solve the equation $\log_7(4x+5) = 0$.
解 $\log_7(4x+5) = 0$。

61. The sound intensity level of piledriver A and piledriver B are 108 dB and 114 dB respectively. How many times is the sound intensity produced by piledriver B to that by piledriver A?
(Give your answer correct to 3 significant figures.)
打樁機 A 和打樁機 B 所產生的聲強級分別是 108 dB 和 114 dB。問打樁機 B 所產生的聲強是打樁機 A 的多少倍?
（答案須準確至三位有效數字。）

62. The sound intensity level of a road is 75 dB measured from the building next to the road. After building noise absorber along the road, the sound intensity level decreases to 65 dB. How many times is the sound intensity of the road before building noise absorber to that after building noise absorber?
一條街道的路面噪音的聲強級是 75 dB。當設置吸音裝置後，路面噪音的聲強級減少至 65 dB。問設置吸音裝置前的路面噪音的聲強是設置吸音裝置後的多少倍？

63. The strengths of earthquakes in city *A* in 1979 and in 1995 were measured 6.5 and 7.2 respectively on the Richter scale. How many times is the strength of the earthquake in city *A* in 1995 to that in 1979? (Give your answer correct to 3 significant figures.)
城市 *A* 於 1979 年和 1995 年的地震強度分別是黎克特制 6.5 級和 7.2 級。問城市 *A* 於 1995 年的地震強度是 1979 年的多少倍？
（答案須準確至三位有效數字。）

***64*.** The sound intensity level of a construction site in the evening is 50 dB, whereas the sound intensity level during the day-time increases by 30 dB. How many times is the sound intensity of the construction site during the day-time to that in the evening?
一個建築工程地盤在黃昏時所產生的聲強級是 50 dB，而在日間所產生的聲強級較黃昏時的增加 30 dB。問該建築工程地盤在日間時的聲強是黃昏時的多少倍？

***65*.** How many times is the strength of an earthquake measured 7 on the Richter scale to that measured 4 on the Richter scale?
問黎克特制 7 級地震的強度是黎克特制 4 級地震的多少倍?

Level 2 Questions
程度 2 題目

1. Simplify and express $\sqrt[6]{x^3}\cdot\dfrac{x^{\frac{2}{5}}}{\sqrt{x^3}}$ with positive indices.

化簡 $\sqrt[6]{x^3}\cdot\dfrac{x^{\frac{2}{5}}}{\sqrt{x^3}}$，並以正指數表示答案。

2. Simplify and express $\dfrac{m^{\frac{1}{4}}n^{\frac{2}{3}}}{m^{\frac{1}{6}}n^{\frac{1}{2}}}\div(mn)^{\frac{1}{3}}$ with positive indices.

化簡 $\dfrac{m^{\frac{1}{4}}n^{\frac{2}{3}}}{m^{\frac{1}{6}}n^{\frac{1}{2}}}\div(mn)^{\frac{1}{3}}$，並以正指數表示答案。

3. Simplify and express $2^{-1}y^{\frac{1}{6}}\div\sqrt[3]{27y^2}\cdot(6y^{-\frac{3}{4}})$ with positive indices.

化簡 $2^{-1}y^{\frac{1}{6}}\div\sqrt[3]{27y^2}\cdot(6y^{-\frac{3}{4}})$，並以正指數表示答案。

4. Simplify and express $\sqrt[4]{a^2b}\cdot(a^{\frac{3}{2}}b^2)^{\frac{1}{3}}\cdot(ab^{-\frac{1}{4}})$ with positive indices.

化簡 $\sqrt[4]{a^2b}\cdot(a^{\frac{3}{2}}b^2)^{\frac{1}{3}}\cdot(ab^{-\frac{1}{4}})$，並以正指數表示答案。

5 Simplify and express $\left(\dfrac{27}{64}ab^3\right)^{-\frac{2}{3}}\cdot(3a^{\frac{1}{3}}b)\div(4a^{-\frac{2}{3}}b)^2$ with positive indices.

化簡 $\left(\dfrac{27}{64}ab^3\right)^{-\frac{2}{3}}\cdot(3a^{\frac{1}{3}}b)\div(4a^{-\frac{2}{3}}b)^2$，並以正指數表示答案。

6. Simplify and express $\dfrac{a^3}{b^{-4}}\cdot(a^{-2}b)\cdot\left(\dfrac{a}{b}\right)^{-1}$ with positive indices.

化簡 $\dfrac{a^3}{b^{-4}}\cdot(a^{-2}b)\cdot\left(\dfrac{a}{b}\right)^{-1}$，並以正指數表示答案。

7. Simplify and express $\sqrt{25x} \div (5x^{-\frac{3}{4}}y)^{-2} \cdot (\sqrt[3]{xy})^4$ with positive indices.

化簡 $\sqrt{25x} \div (5x^{-\frac{3}{4}}y)^{-2} \cdot (\sqrt[3]{xy})^4$ ，並以正指數表示答案。

8. Simplify and express $\frac{2b}{\sqrt[4]{a^4b^2}} \cdot \sqrt{a^{\frac{1}{4}}b^3} \cdot (\sqrt[3]{8a^6})^{-1}$ with positive indices.

化簡 $\frac{2b}{\sqrt[4]{a^4b^2}} \cdot \sqrt{a^{\frac{1}{4}}b^3} \cdot (\sqrt[3]{8a^6})^{-1}$ ，並以正指數表示答案。

9. Simplify and express $\sqrt[4]{81x^{-2}y^8} \cdot \frac{3x}{\sqrt{xy^{\frac{2}{3}}}} \cdot \left(\frac{x^{-5}y}{32}\right)^{\frac{2}{5}}$ with positive indices.

化簡 $\sqrt[4]{81x^{-2}y^8} \cdot \frac{3x}{\sqrt{xy^{\frac{2}{3}}}} \cdot \left(\frac{x^{-5}y}{32}\right)^{\frac{2}{5}}$ ，並以正指數表示答案。

10. Simplify and express $(16m^4n)^{\frac{1}{2}} \div \left(\frac{1}{\sqrt[3]{mn^2}}\right)^{-\frac{3}{4}} \div (2m^{\frac{1}{4}}n^{-3})$ with positive indices.

化簡 $(16m^4n)^{\frac{1}{2}} \div \left(\frac{1}{\sqrt[3]{mn^2}}\right)^{-\frac{3}{4}} \div (2m^{\frac{1}{4}}n^{-3})$ ，並以正指數表示答案。

11 Solve the equation $3^{2x} + 8(3^x) - 9 = 0$.

解 $3^{2x} + 8(3^x) - 9 = 0$ 。

12. Solve the equation $2^{x+2} - 2^{x+1} + 2^{x-1} = 40$.

解 $2^{x+2} - 2^{x+1} + 2^{x-1} = 40$ 。

13. Solve the equation $2(5^{x+1}) - 3(5^x) - 5^{x-1} = 34$.

解 $2(5^{x+1}) - 3(5^x) - 5^{x-1} = 34$ 。

14. Solve the equation $4(4^{2x}) - 17(4^x) + 4 = 0$.

解 $4(4^{2x}) - 17(4^x) + 4 = 0$ 。

***15*.** Solve the simultaneous equations $\begin{cases} 5^{3x+2y} = 25 \\ 3^{2x+y} = 81 \end{cases}$.

解聯立方程 $\begin{cases} 5^{3x+2y} = 25 \\ 3^{2x+y} = 81 \end{cases}$。

***16*.** Solve the simultaneous equations $\begin{cases} 6^{x+y} = 216 \\ 10^{2x-y} = 1 \end{cases}$.

解聯立方程 $\begin{cases} 6^{x+y} = 216 \\ 10^{2x-y} = 1 \end{cases}$。

17. Solve the simultaneous equations $\begin{cases} 2^{x+3y} = \dfrac{1}{16} \\ 4^x - 2(8^y) = 0 \end{cases}$.

解聯立方程 $\begin{cases} 2^{x+3y} = \dfrac{1}{16} \\ 4^x - 2(8^y) = 0 \end{cases}$。

18. Solve the simultaneous equations $\begin{cases} 3^{2x} - 3^{y-1} = 0 \\ 4(2^{x+y}) = 64 \end{cases}$.

解聯立方程 $\begin{cases} 3^{2x} - 3^{y-1} = 0 \\ 4(2^{x+y}) = 64 \end{cases}$。

19. Find the value of $3\log 2 - \log\left(\dfrac{1}{125}\right)$ without using a calculator.

試不使用計算機，求 $3\log 2 - \log\left(\dfrac{1}{125}\right)$ 的值。

20. Find the value of $\dfrac{1}{3}\log_2 27 + \log_2 12 - 2\log_2 \dfrac{3}{4}$ without using a calculator.

試不使用計算機，求 $\dfrac{1}{3}\log_2 27 + \log_2 12 - 2\log_2 \dfrac{3}{4}$ 的值。

21. Find the value of $\dfrac{\log_a \sqrt[3]{216}+\dfrac{1}{4}\log_a 36}{\log_a\left(\dfrac{1}{9}\right)-2\log_a 2}$ without using a calculator.

試不使用計算機，求 $\dfrac{\log_a \sqrt[3]{216}+\dfrac{1}{4}\log_a 36}{\log_a\left(\dfrac{1}{9}\right)-2\log_a 2}$ 的值。

22. Find the value of $\dfrac{-2\log 27+2.5\log 9}{\dfrac{1}{2}\log 18-\log\left(\dfrac{1}{\sqrt{50}}\right)-1}$ without using a calculator.

試不使用計算機，求 $\dfrac{-2\log 27+2.5\log 9}{\dfrac{1}{2}\log 18-\log\left(\dfrac{1}{\sqrt{50}}\right)-1}$ 的值。

23. Simplify $\dfrac{\dfrac{1}{3}\log x^2y^3-\log x^3y}{\log\sqrt[3]{x}}$, where $x > 0$, $y > 0$ and $x, y \neq 1$.

化簡 $\dfrac{\dfrac{1}{3}\log x^2y^3-\log x^3y}{\log\sqrt[3]{x}}$，其中 $x > 0$，$y > 0$，且 x，$y \neq 1$。

24. Simplify $\log_3\left(\sqrt{\dfrac{9^{3x+1}}{27^{2x-y}}}\right)$, where $x > 0$, $y > 0$ and $x, y \neq 1$.

化簡 $\log_3\left(\sqrt{\dfrac{9^{3x+1}}{27^{2x-y}}}\right)$，其中 $x > 0$，$y > 0$，且 x，$y \neq 1$。

25. Simplify $\dfrac{3\log x^4+6\log y}{2\log\left(\dfrac{x}{y}\right)+\log y^3}$, where $x > 0$, $y > 0$ and $x, y \neq 1$.

化簡 $\dfrac{3\log x^4+6\log y}{2\log\left(\dfrac{x}{y}\right)+\log y^3}$，其中 $x > 0$，$y > 0$，且 x，$y \neq 1$。

26. Simplify $\dfrac{\log\sqrt{xy^3}-\log xy}{-\frac{3}{2}\log xy+\log x^2y}$, where $x > 0$, $y > 0$ and $x, y \neq 1$.

化簡 $\dfrac{\log\sqrt{xy^3}-\log xy}{-\frac{3}{2}\log xy+\log x^2y}$，其中 $x > 0$，$y > 0$，且 x，$y \neq 1$。

27. Given that $\log 5 = x$ and $\log 6 = y$, express $\log 15$ in terms of x and y.
已知 $\log 5 = x$ 及 $\log 6 = y$，試以 x 和 y 表示 $\log 15$。

28. Given that $\log 4 = x$ and $\log 24 = y$, express $\log 6$ in terms of x and y.
已知 $\log 4 = x$ 及 $\log 24 = y$，試以 x 和 y 表示 $\log 6$。

29. Solve the equation $\log(2x-1)+\log 4 = 2$.
解 $\log(2x-1)+\log 4 = 2$。

30. Given that $\log 4 = x$ and $\log 24 = y$, express $\log 3$ in terms of x and y.
已知 $\log 4 = x$ 及 $\log 24 = y$，試以 x 和 y 表示 $\log 3$。

31. Given that $\log 5 = x$ and $\log 6 = y$, express $\log\sqrt{288}$ in terms of x and y.
已知 $\log 5 = x$ 及 $\log 6 = y$，試以 x 和 y 表示 $\log\sqrt{288}$。

32. Solve the equation $\log_2(3x+2)-3\log_2 4 = -3$.
解 $\log_2(3x+2)-3\log_2 4 = -3$。

33. Solve the equation $[\log_3(4x-3)]^2-3\log_3(4x-3)-4 = 0$.
解 $[\log_3(4x-3)]^2-3\log_3(4x-3)-4 = 0$。

34 Solve the equation $\log(x+1)+\log(x+4)=\dfrac{2}{3}\log(10\sqrt{10})$.

解 $\log(x+1)+\log(x+4)=\dfrac{2}{3}\log(10\sqrt{10})$。

35. Solve the simultaneous equations $\begin{cases}x+2y=8\\ \log x=2\log y+1\end{cases}$.

解聯立方程 $\begin{cases}x+2y=8\\ \log x=2\log y+1\end{cases}$。

36. Solve the simultaneous equations $\begin{cases} 7^{3x-y}=1 \\ \log(2x+y)-2\log\sqrt[3]{8}=1 \end{cases}$.

解聯立方程 $\begin{cases} 7^{3x-y}=1 \\ \log(2x+y)-2\log\sqrt[3]{8}=1 \end{cases}$。

37. Solve the simultaneous equations $\begin{cases} \log(x-y)=0 \\ 2\log_4\sqrt{x}+\log_4 y=1+\log_4 3 \end{cases}$.

解聯立方程 $\begin{cases} \log(x-y)=0 \\ 2\log_4\sqrt{x}+\log_4 y=1+\log_4 3 \end{cases}$。

38. Solve the simultaneous equations $\begin{cases} 3(9^{6-x})-3^{2y+1}=0 \\ \log_3 xy^4=2+3\log_3 y \end{cases}$.

解聯立方程 $\begin{cases} 3(9^{6-x})-3^{2y+1}=0 \\ \log_3 xy^4=2+3\log_3 y \end{cases}$。

39. The sound intensity level of a busy road is 80 dB. During traffic jam, the sound intensity of traffic on the road is reduced to 0.1% of the original. Find the sound intensity level of the road during traffic jam.
一條繁忙的街道的路面噪音的聲強級是 80 dB。當交通擠塞時，路面所產生的聲強比平時減少 0.1%。求交通擠塞時該條街道的路面噪音的聲強級。

***40*.** The sound intensity level in a gathering is 45 dB. After playing music, the sound intensity doubles. What is the sound intensity level after playing music?
(Give your answer correct to 3 significant figures.)
一次聚會開始時所產生的聲強級是 45 dB。當音樂播放後，聲強高了一倍。求播放音樂後所產生的聲強級。
（答案須準確至三位有效數字。）

41. If the strength of an earthquake is $\frac{1}{5}$ times to that measured 8.8 on the Richter scale, what is the magnitude of the required earthquake on the Richter scale?
(Give your answer correct to 3 significant figures.)

若一地震的強度是黎克特制 8.8 級的 $\frac{1}{5}$ 倍，那麼該地震的強度相等於黎克特制多少級呢？
（答案須準確至三位有效數字。）

42 If the sound intensity in a construction site is increased by 10%, what is the increase of the corresponding sound intensity level?
(Give your answer correct to 3 significant figures.)
若一個建築工程地盤的聲強增加 10%，則相應的聲強級會增加多少？
（答案須準確至三位有效數字。）

43. If the strength of an earthquake is 10 times to that measured 4.3 on the Richter scale, what is the magnitude of the required earthquake on the Richter scale?
若一地震的強度是黎克特制 4.3 級的 10 倍，那麼該地震的強度相等於黎克特制多少級呢？

Level 2+ Questions
程度 2+ 題目

1 **(a)** It is given that $f(x) = x^4 - 5x^3 + 20x - 16$.

已知 $f(x) = x^4 - 5x^3 + 20x - 16$。

(i) Find the values of $f(4)$, $f(6)$ and $f(10)$.

求 $f(4)$、$f(6)$ 和 $f(10)$ 的值。

(ii) Hence, factorize $f(x)$ completely.

由此，因式分解 $f(x)$。

(b) Given that $\log 2 = a$ and $\log 3 = b$. By using the above results, express the following in terms of a and b.

已知 $\log 2 = a$ 及 $\log 3 = b$。試利用以上的結果，以 a 和 b 表示下列各對數的值。

(i) $\log 3200$ **(ii)** $\log 518.4$

2 Given that $\frac{xy}{z} = 32$. If $\frac{\log_2 x}{1} = \frac{\log_2 y}{2} = \frac{\log_2 z}{-2} = k$, find

已知 $\frac{xy}{z} = 32$。若 $\frac{\log_2 x}{1} = \frac{\log_2 y}{2} = \frac{\log_2 z}{-2} = k$，求

(a) the value of k,

k 的值；

(b) the values of x, y and z.

x、y 和 z 的值。

3. (a) Given that the equation $\frac{\log_2 a \log_2 x}{\log_2 \frac{x}{2}} - \frac{\log_2 b}{\log_2 x^2} = 0$, where $a \neq 1$ and $b \neq 1$.

已知方程 $\frac{\log_2 a \log_2 x}{\log_2 \frac{x}{2}} - \frac{\log_2 b}{\log_2 x^2} = 0$，其中 $a \neq 1$ 及 $b \neq 1$。

(i) Set up a quadratic equation in $\log_2 x$.

建立一個以 $\log_2 x$ 為變數的二次方程。

(ii) If the equation has a double real root, express b in terms of a.

若該方程有一個二重實根，試以 a 表示 b。

(b) Hence, solve $\frac{\log_2 x}{2\log_2 \frac{x}{2}} - \frac{4}{\log_2 x^2} = 0$.

由此，解 $\dfrac{\log_2 x}{2\log_2 \dfrac{x}{2}} - \dfrac{4}{\log_2 x^2} = 0$ 。

4. Given that $a = \log\left(1 - \dfrac{1}{9}\right)$ and $b = \log\left(1 - \dfrac{1}{81}\right)$.

已知 $a = \log\left(1 - \dfrac{1}{9}\right)$ 及 $b = \log\left(1 - \dfrac{1}{81}\right)$ 。

(a) Show that $3\log 2 = 2\log 3 + a = 4\log 3 - 1 + b$.

證明 $3\log 2 = 2\log 3 + a = 4\log 3 - 1 + b$ 。

(b) Express the following in terms of *a* and *b*.

試以 a 和 b 表示下列各題。

(i) log 3 **(ii)** log 2 **(iii)** log 72

5. Solve the following simultaneous equations.

解下列各聯立方程。

(a) $\begin{cases} 3^x + 4^y = 43 \\ 3^{x-1} + 2^{2y+1} = 41 \end{cases}$ **(b)** $\begin{cases} 5^{x-y} = 0.0016 \\ \log_{\sqrt{3}}(6x + 33) - 2 = 2\log_{\sqrt{3}} y \end{cases}$

6. (a) Let $T_n = x^n + \dfrac{1}{x^n}$, where *n* is a positive integer. Prove that $T_n = T_{n-1} \cdot T_1 - T_{n-2}$ for $n \geq 3$.

設 $T_n = x^n + \dfrac{1}{x^n}$ ，其中 n 是一個正整數。證明當 $n \geq 3$ 時， $T_n = T_{n-1} \cdot T_1 - T_{n-2}$ 。

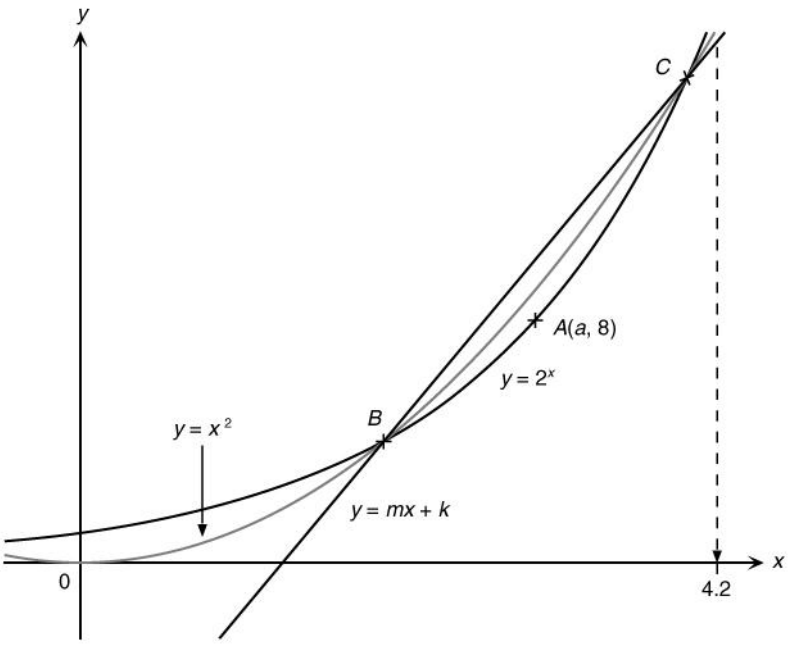
(b) Given that $T_1 = 4$. Hence, or otherwise, find the values of T_2, T_3 and T_4.

已知 $T_1 = 4$ 。由此，或用其他方法，求 T_2、T_3 及 T_4 的值。

7 Solve $9\sqrt{x+1}\left(\sqrt[4]{x+1} - \dfrac{x}{9}\right) = \sqrt{x+1} + 8$.

解 $9\sqrt{x+1}\left(\sqrt[4]{x+1} - \dfrac{x}{9}\right) = \sqrt{x+1} + 8$ 。

8. The figure shows the graphs of $y = 2^x$, $y = x^2$ and $y = mx + k$. $A(a, 8)$ lies on the graph of $y = 2^x$. The three graphs intersect at B and C respectively.
下圖所示為 $y = 2^x$，$y = x^2$ 及 $y = mx + k$ 的圖像。$A(a, 8)$ 位於 $y = 2^x$ 的圖像上。該三個圖像分別相交於 B 和 C。



(a) **(i)** Find the coordinates of A, B and C.
求 A、B 和 C 的坐標。

(ii) Find the values of m and k.
求 m 和 k 的值。

(b) Solve $x^2 - 6x + 8 \le 0$ graphically.
利用圖解法解 $x^2 - 6x + 8 \le 0$。

(c) For $0 \le x \le 4.2$, find the range of possible values of x that satisfies $x^2 \ge 2^x$.
對於 $0 \le x \le 4.2$，求 x 值的可能範圍，使 $x^2 \ge 2^x$。

Multiple Choice Questions

多項選擇題

1. $(x^{3n})^2 =$

A. x^{6n}

B. x^{9n}

C. x^{6n^2}

D. x^{9n^2}

2 $\sqrt{\sqrt{\sqrt{\sqrt{x^{16}}}}} =$

A. 1

B. x

C. x^2

D. x^4

3. $\dfrac{a^{\frac{3}{2}}(\sqrt[6]{a})^5}{\sqrt[3]{a^{-2}}} =$

A. $a^{\frac{5}{3}}$

B. $a^{\frac{5}{6}}$

C. a^3

D. a^6

4. $\dfrac{(a^4b^{-2})^3}{a^{-2}b} =$

A. $\dfrac{a^{10}}{b^7}$

B. $\dfrac{a^{10}}{b^5}$

C. $\dfrac{a^{14}}{b^7}$

D. $\dfrac{a^{14}}{b^5}$

5. Solve $16^x - 3(4^x) - 4 = 0$.

解 $16^x - 3(4^x) - 4 = 0$。

A. $x = 1$

B. $x = 0$ or $\dfrac{1}{4}$

C. $x = 0$ or 1

D. $x = \dfrac{1}{4}$ or 1

6. If $4(8^{x-1}) = \sqrt[3]{2^{x-2}}$, then $x =$

若 $4(8^{x-1}) = \sqrt[3]{2^{x-2}}$，則 $x =$

A. -8.

B. $-\dfrac{5}{8}$.

C. $\dfrac{1}{8}$.

D. 5.

7. If $3^x = 36 - 3^{x-1}$, then $x =$

若 $3^x = 36 - 3^{x-1}$，則 $x =$

A. 1.

B. 2.

C. 3.

D. 4.

$\therefore \quad x = 3$

8. Solve $\begin{cases} 5^{x+2y}=\dfrac{1}{25} \\ 2^{x-3}=\sqrt[5]{32}(4^{y}) \end{cases}$.

解聯立方程 $\begin{cases} 5^{x+2y}=\dfrac{1}{25} \\ 2^{x-3}=\sqrt[5]{32}(4^{y}) \end{cases}$。

A. $x=-2, y=0$

B. $x=-1, y=-\dfrac{1}{2}$

C. $x=1, y=-\dfrac{3}{2}$

D. $x=2, y=-2$

9. $\log_{2\sqrt{2}} 64=$

A. 1

B. 2

C. 4

D. 8

10. If $\log b-2\log a=1$, then $b=$

若 $\log b-2\log a=1$，則 $b=$

A. $10a^2$.

B. $10+a^2$.

C. $10+2a$.

D. $1+2a$.

11 $\dfrac{\log a+2\log b}{\log(ab)-\log\sqrt{a}}=$

A. $\log(b\sqrt{a})$

B. $b\sqrt{a}$

C. 2

D. $\dfrac{1}{2}$

***12*.** If $3=10^{a}$ and $4=10^{b}$, then $\log\dfrac{5}{6}=$

若 $3=10^{a}$ 及 $4=10^{b}$，則 $\log\dfrac{5}{6}=$

A. $1+a+b$.

B. $1-a+b$.

C. $1-a-b$.

D. $1+a-b$.

***13*.** Solve $\log_2(x-1)=2-\log_2(x+2)$.

解 $\log_2(x-1)=2-\log_2(x+2)$。

A. $x=2$

B. $x=3$

C. $x=-3$ or 2

D. $x=-2$ or 3

14. If $9^{x}=12$, then $x=$

若 $9^{x}=12$，則 $x=$

A. $\dfrac{\log 4}{\log 3}$.

B. $\log\dfrac{4}{3}$.

C. $\dfrac{\log 12}{\log 9}$.

D. $\dfrac{1}{9}\log 12$.

15. If $\log\sqrt[3]{x}=k$, then $\log\left(\dfrac{1}{x^2}\right)=$

若 $\log\sqrt[3]{x}=k$，則 $\log\left(\dfrac{1}{x^2}\right)=$

A. $-6k$

B. $-9k^2$

C. $\dfrac{1}{6k}$

D. $\dfrac{1}{9k^2}$

16. Solve $2(\log x)^2 - \log x^3 - 2 = 0$.

解 $2(\log x)^2 - \log x^3 - 2 = 0$ 。

A. $x = \frac{1}{\sqrt{10}}$ or 100

B. $x = \frac{1}{100}$ or $\sqrt{10}$

C. $x = -\sqrt{10}$ or 100

D. $x = \sqrt{10}$ or -100

17. Solve $\begin{cases} 3^{x-1}2^x = 2(6^y) \\ \log_3(x-2) - \log_3 y + 1 = 0 \end{cases}$.

解 $\begin{cases} 3^{x-1}2^x = 2(6^y) \\ \log_3(x-2) - \log_3 y + 1 = 0 \end{cases}$ 。

A. $x = \frac{3}{2}, y = \frac{5}{2}$

B. $x = -\frac{3}{2}, y = \frac{5}{2}$

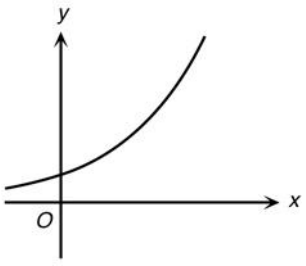
C. $x = \frac{5}{2}, y = \frac{3}{2}$

D. $x = \frac{5}{2}, y = -\frac{3}{2}$

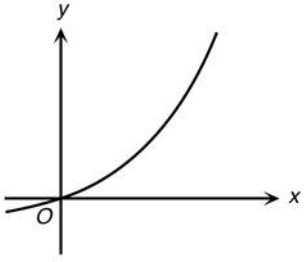
18. Which of the following may represent the graph of $y = 2^x$?

下列何者能代表 $y = 2^x$ 的圖像？

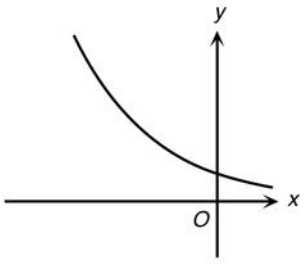
A.



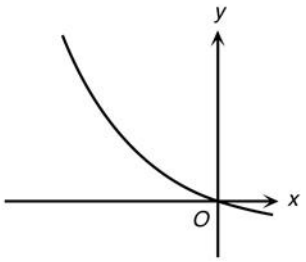
B.



C.



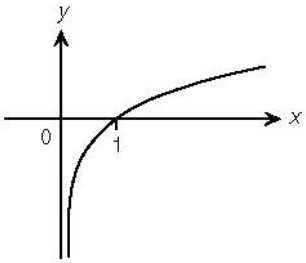
D.



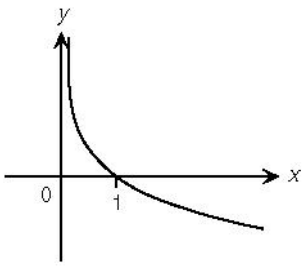
19. Which of the following MUST not be a graph of $y = \log_a x$, where $a > 0$ and $a \neq 1$?

下列何者**不可能**是 $y = \log_a x$ 的圖像？（其中 $a > 0$ 及 $a \neq 1$。）

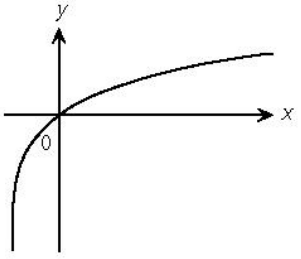
I.



II.



III.



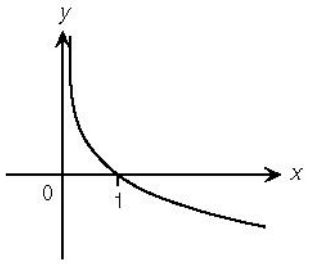
A. II only 只有 II

B. III only 只有 III

C. I and II only 只有 I 及 II

D. II and III only 只有 II 及 III

20. Which of the following functions is represented by the graph shown below?
下列哪一個函數可由以下圖像所代表？



A. $y = \log_2 x$

B. $y = \log_{0.5} x$

C. $y = 2^x$

D. $y = \left(\frac{1}{2}\right)^x$