3 Approximations

A. Unit Conversion

The following table shows some common unit conversion.

Quantity	Example
Time	1 minute (min) = 60 seconds (s)
	1 hour (h) = 60 minutes (min)
	1 day = 24 hours (h)
Weight	1 g = 1000 mg
	1 kg = 1000 g
Length	1 cm = 10 mm
	1 m = 100 cm
	1 km = 1000 m
Capacity	1 mL = 1 cm^3
	1 L = 1000 mL
	1 L = 1000 cm^3
	1 m^3 = 1000 L

Example 1

Express 60 cm in the following units.

(a) mm

(b) m

Solution

(a) 60 cm

$= 60 \times 10$ mm

$= \underline{600 \text{ mm}}$

(b) 60 cm

$= \frac{60}{100}$ m

$= \underline{0.6 \text{ m}}$

Example 2

Express 75 min in the following units.

(a) s

(b) h

Solution

(a) 75 min
$= 75 \times 60$ s
$= \underline{4500 \text{ s}}$

(b) 75 min
$= \frac{75}{60}$ h
$= \underline{1.25 \text{ h}}$

B. Approximate Value

Mathematical term	Example
Exact value	7.56 is the exact value of 2.1×3.6.
Approximate value	7.6 is an approximate value of 7.56.

Rounding off is a method of finding an approximate value.

Take 17 825.364 as an example. It can be rounded off as follows:

Approximate value	Degree of accuracy
20 000	correct to the nearest ten thousand
18 000	correct to the nearest thousand
17 800	correct to the nearest hundred
17 830	correct to the nearest ten
17 825	correct to the nearest one
17 825.4	correct to the nearest tenth or correct to 1 decimal place
17 825.36	correct to the nearest hundredth or correct to 2 decimal plac

Example 3

(a) Round off 98 467 to the nearest thousand.

(b) Round off 4.98 to 1 decimal place.

Solution

(a) 98 467 = 98 000 (correct to the nearest thousand)

(b) 4.98 = 5.0 (correct to 1 decimal place)

Example 4

(a) Round off 46.95 kg to the nearest kg.

(b) Round off 46.95 kg to the nearest 0.1 kg.

Solution

(a) 46.95 kg = 47 kg (correct to the nearest kg)

(b) 46.95 kg = 47.0 kg (correct to the nearest 0.1 kg)

Example 5

Express the following fractions in decimals and round off your results to 2 decimal places.

(a) $\frac{1}{16}$ **(b)** $\frac{7}{8}$

Solution

a) $\frac{1}{16} = 0.0625$

$= 0.06$ (correct to 2 decimal places)

) $\frac{7}{8} = 0.875$

$= 0.88$ (correct to 2 decimal places)

C. Solving Problems by Approximations

When handling problems involving numbers in daily life, we may use approximate values to do calculations to get a result close to the exact value.

For example, if we have \$45 and each piece of cake costs \$4.8, about how many pieces of cake can we buy?

Firstly, we can round off the cost of a piece of cake to the nearest dollar, i.e. \$5. Then, by computing $\frac{45}{5}$, we get 9. Therefore, we can buy about 9 pieces of cake. This is simpler than finding the exact value $\left(\text{i.e. } \frac{45}{4.8}\right)$.

Example 6

There are 596 students joining the school picnic. If one coach can carry 40 students, estimate the number of coaches required to carry all the students.

Solution

There are about 600 students joining the school picnic.

$\therefore$ The required number of coaches

$$\approx \frac{600}{40}$$

◀ The symbol '$\approx$' means 'approximately equal to'.

$$= \underline{\underline{15}}$$

Example 7

Calvin walks 49 minutes from one end of a sidewalk to the other end. If he walks about 81.5 m in one minute, estimate the length of the sidewalk in km.

Solution

Length of the sidewalk

$= 49 \times 81.5 \text{ m}$

$\approx 50 \times 80 \text{ m}$

$= 4000 \text{ m}$

$= \frac{4000}{1000} \text{ km}$

$= \underline{\underline{4 \text{ km}}}$

Key Terms / Phrases

Pronunciation

unit conversion	單位轉換	rounding off	四捨五入法	estimate	估算
exact value	真確值	correct to	捨入至 / 準確至		
approximate value	近似值	degree of accuracy	準確度		

Useful Sentences

Round off the following numbers to 1 decimal place.	把下列各數捨入至一位小數。
Estimate the average number of students in a school.	估算每間學校的平均學生人數。
Estimate how many books Harry can buy.	估算思朗可買多少本書。
Estimate whether Amy can afford to buy 12 dresses.	估算雅文能否購買 12 條裙子。
The symbol '≈' means approximately equal to.	符號「≈」表示大約等於。

Exercise 3

1. Complete the following unit conversions.

(a) 100 min = _____ s

(b) 5 h = _____ min

(c) 1800 s = _____ h

(d) 1200 g = _____ kg

(e) 0.85 L = _____ mL

(f) 6 L = _____ cm^3

(g) 3.5 m^3 = _____ L

(h) 150 m = _____ cm

(i) 10 m = _____ mm

(j) 0.5 km = _____ cm

2. **In each of the following, determine whether the underlined number** is an exact value or an **approximate value. Put a '✓' in the appropriate box.**

	Exact value	Approximate value
(a) **5 kg equals 5000 g.**	☐	☐
(b) **Number of *passengers* of the train is around 900.**	☐	☐
(c) **The price of a shirt is $58.**	☐	☐
(d) There are about 1200 students in the school.	☐	☐

passenger 乘客

3. Round off the following numbers to the required degree of accuracy.

Number	8637	742 218	37 425	99 999
Correct to the nearest ten thousand				
Correct to the nearest thousand				
Correct to the nearest hundred				
Correct to the nearest ten				

4. Round off the following numbers to the required degree of accuracy.

Number	3.4637	0.3389	6.406 92	0.9999
Correct to the nearest one				
Correct to 1 decimal place				
Correct to 2 decimal places				
Correct to 3 decimal places				

5. Round off the following fractions to the nearest one.

Fraction	$\frac{2}{3}$	$3\frac{1}{3}$	$1\frac{5}{8}$	$8\frac{7}{15}$	$\frac{15}{4}$
Correct to the nearest one					

6. **(a)** Round off 7.45 m to the nearest m. _____

(b) Round off 496 mL to the nearest 10 mL. _____

(c) Round off \$163.89 to the nearest \$0.1. _____

(d) Round off 7649 g to the nearest kg. _____

7. **Round off each of the numbers in the following expressions to the nearest one first, and then find th approximate values of the expressions.**

(a) 3.2+2.7 _____ **(b)** 15.17−9.85 _____

(c) 7.06×3.9 _____ **(d)** 19.8÷10.4 _____

8. **Round off each of the numbers in the following expressions to the nearest hundred first, and then find the approximate values of the expressions.**

(a) 2283−1098 _____ **(b)** 113+202−97 _____

(c) 189×101 _____ **(d)** 999÷218 _____

9. 8 pieces of toys cost $117. Estimate the cost of one piece of toy. ______________

10. A box weighs 59.7 kg. Estimate the total weight of 3 boxes. ______________

Solve the following problems. Show your working steps clearly. **(11 – 15)**

11. If Raymond can type 38 English words in one minute, estimate the number of words that can be typed in two hours.

12. In a theatre, there are 89 seats in each row. If there are 71 rows in the theatre, about how many seats are there in the theatre?

13. **A major shopping centre has approximately 49 780 visitors per day. Estimate the total number of visitors per month.**

14. Polly has \$1605 as savings. If she spends \$80 every day, estimate the number of days needed for her to use up the savings.

15. The costs of photo frames A, B and C are \$40.2, \$69.6 and \$109.7 respectively. Estimate the total cost of 2 pieces of A, 3 pieces of B and 1 piece of C.