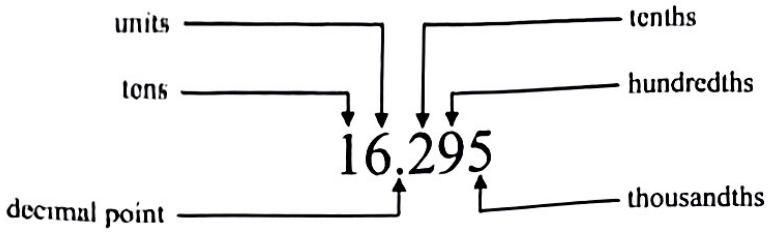
2 Decimals and Fractions

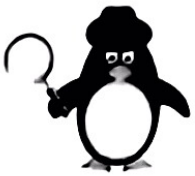
A. Conversion between Decimals and Fractions

16.295 is a decimal, which can be read as sixteen point two nine five.



Fractions can be classified into following three types:

Proper fraction	Improper fraction	Mixed fraction
The numerator is smaller than the denominator.	The numerator is greater than or equal to the denominator.	An improper fraction which is written as a sum of a natural number and a proper fraction.
numerator → 3, denominator → 5	$\frac{7}{4}$	integral part → $6\frac{2}{3}$ ← fractional part
Read as : three-fifths / three over five	Read as : seven quarters / seven over four	Read as : six and two-thirds / six and two over three



Improper fractions can be converted into mixed fractions and vice versa.

e.g. $\frac{7}{4} = 1\frac{3}{4}$

Example 1

Convert 0.45 into a fraction.

Solution

$$0.45 = \frac{\cancel{45}^{9}}{\cancel{100}_{20}}$$

$$= \frac{9}{20}$$

◀ Reduce the fraction to its simplest form.

Example 2

Convert $\frac{3}{5}$ and $\frac{7}{25}$ into decimals.

Solution

Method 1: Expand the fractions

$$\frac{3}{5}=\frac{3\times 2}{5\times 2}$$
$$=\frac{6}{10}$$
$$=\underline{\underline{0.6}}$$

$$\frac{7}{25}=\frac{7\times 4}{25\times 4}$$
$$=\frac{28}{100}$$
$$=\underline{\underline{0.28}}$$

Method 2: Use division

$$\frac{3}{5}=3\div 5$$
$$=\underline{\underline{0.6}}$$

$$\begin{array}{r} 0.6 \\ 5\overline{)3.0} \\ \underline{3\ 0} \end{array}$$

$$\frac{7}{25}=7\div 25$$
$$=\underline{\underline{0.28}}$$

$$\begin{array}{r} 0.28 \\ 25\overline{)7.00} \\ \underline{5\ 0} \\ 2\ 00 \\ \underline{2\ 00} \end{array}$$

Example 3

Arrange the following numbers in descending order.

$$1\frac{1}{4},\ 1\frac{3}{8},\ 1.125$$

Solution

Method 1: Convert the numbers into fractions

$$1\frac{1}{4}=1\frac{1\times 2}{4\times 2}=1\frac{2}{8}$$
$$1.125=1\frac{125}{1000}=1\frac{1}{8}$$
$$\therefore\ 1\frac{3}{8}>1\frac{2}{8}>1\frac{1}{8}$$ ◀ **Compare the numerators.**
$$\therefore\ 1\frac{3}{8}>1\frac{1}{4}>1.125$$

Method 2: Convert the numbers into decimals

$$1\frac{1}{4}=1+\frac{1}{4}=1+0.25=1.25$$
$$1\frac{3}{8}=1+\frac{3}{8}=1+0.375=1.375$$
$$\because\ 1.375>1.25>1.125$$
$$\therefore\ 1\frac{3}{8}>1\frac{1}{4}>1.125$$

B. Basic Operations of Decimals and Fractions

The following table shows some basic arithmetic operations of decimals.

Addition	**Subtraction**
e.g. $2.08+3.42$ $=\underline{\underline{5.5}}$ $\begin{array}{r} 2.08 \\ +\ 3.42 \\ \hline 5.50 \end{array}$	e.g. $5.4-2.25$ $=\underline{\underline{3.15}}$ $\begin{array}{r} 5.40 \\ -\ 2.25 \\ \hline 3.15 \end{array}$
Multiplication	**Division**
e.g. 8.12×3.4 $=\underline{\underline{27.608}}$ $\begin{array}{r} 8.12 \\ \times\ \ 3.4 \\ \hline 24\,360 \\ +\ 3\,248 \\ \hline 27.608 \end{array}$	e.g. $3.5\div0.4$ $=(3.5\times10)\div(0.4\times10)$ $=35\div4$ $=\underline{\underline{8.75}}$ $\begin{array}{r} 8.75 \\ 4\overline{)\,35.00} \\ 32 \\ \hline 30 \\ 28 \\ \hline 20 \\ 20 \\ \hline \end{array}$

Example 4

Evaluate $2.5\times3.6\div1.5$.

Solution

$2.5\times3.6\div1.5=9\div1.5$

$=(9\times10)\div(1.5\times10)$

$=90\div15$

$=\underline{\underline{6}}$

$\begin{array}{r} 2.5 \\ \times\ 3.6 \\ \hline 7\,50 \\ +\ 1_{1}50 \\ \hline 9.00 \end{array}$

◀ Multiply both the divisor and the dividend by 10 so that the divisor becomes a whole number.

The following table shows some basic arithmetic operations of fractions.

Addition	**Subtraction**
e.g. $\frac{1}{3}+\frac{1}{6}=\frac{2}{6}+\frac{1}{6}$ $=\frac{3}{6}$ $=\underline{\underline{\frac{1}{2}}}$	e.g. $3\frac{2}{5}-2\frac{1}{15}=3\frac{6}{15}-2\frac{1}{15}$ $=1\frac{5}{15}$ $=\underline{\underline{1\frac{1}{3}}}$
Multiplication	**Division**
e.g. $3\frac{1}{3}\times\frac{1}{15}=\frac{\overset{2}{\cancel{10}}}{3}\times\frac{1}{\cancel{15}_{3}}$ $=\underline{\underline{\frac{2}{9}}}$	e.g. $\frac{5}{27}\div\frac{2}{9}=\frac{5}{\cancel{27}_{3}}\times\frac{\cancel{9}^{1}}{2}$ $=\underline{\underline{\frac{5}{6}}}$

If the multiplication and division of fractions involve mixed fractions, we should first change the mixed fractions into improper fractions.

Example 5

Evaluate $2\frac{1}{4}+3\frac{3}{4}\div\frac{5}{6}$.

Solution

$$2\frac{1}{4}+3\frac{3}{4}\div\frac{5}{6}=2\frac{1}{4}+\frac{15}{4}\times\frac{6}{5}$$

◀ Express $3\frac{3}{4}$ as an improper fraction.

$$=2\frac{1}{4}+\frac{9}{2}$$

◀ Perform multiplication and division before addition and subtraction.

$$=2\frac{1}{4}+4\frac{1}{2}$$

$$=2\frac{1}{4}+4\frac{2}{4}$$

◀ Express the fractions in a common denominator.

$$=6\frac{3}{4}$$

Example 6

Harry bought a $\frac{3}{4}$ kg chocolate cake and ate $\frac{3}{5}$ of it. Find the weight of the remaining chocolate cake.

Solution

Weight of the remaining chocolate cake

$$=\frac{3}{4}\times\left(1-\frac{3}{5}\right)\text{ kg}$$

$$=\frac{3}{4}\times\frac{2}{5}\text{ kg}$$

$$=\frac{3}{10}\text{ kg}$$

Example 7

The selling price of a pen is \$6.3 and the selling price of a pencil is \$3.7. What is the total selling price of half a dozen pens and $1\frac{1}{3}$ dozen pencils?

Solution

Total selling price

$$= \$\left(6.3\times12\times\frac{1}{2}+3.7\times12\times1\frac{1}{3}\right)$$

$$= \$\left(6.3\times6+3.7\times12\times\frac{4}{3}\right)$$

$$= \$(6.3\times6+3.7\times16)$$

$$= \$(37.8+59.2)$$

$$= \underline{\underline{\$97}}$$

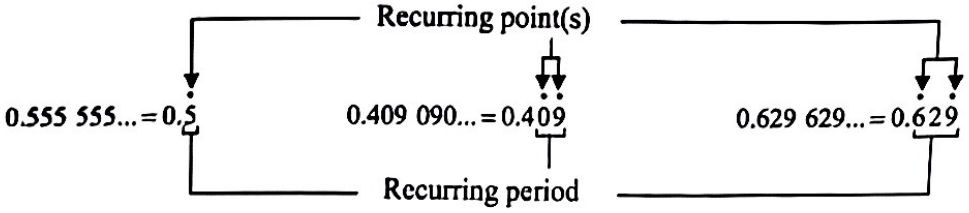
Knowing More

Recurring Decimals

In a decimal, if a digit or a pattern of digits after the decimal point repeats continuously, the decimal is called a recurring decimal. For example: 0.555..., 0.**409** 09 ..., 0.**629** 629... are recurring decimals.

In a recurring decimal, the repeated part is called the recurring period. It is indicated by the recurring point(s).

For example:



We can use recurring decimals to represent fractions as follows:

$\frac{1}{3}=0.333...=0.\dot{3}$, $\frac{8}{11}=0.7272...=0.\dot{7}\dot{2}$, $\frac{11}{27}=0.407\ 407...=0.\dot{4}0\dot{7}$

Key Terms / Phrases

Pronunciation

decimal	小數	thousandths	千分位	mixed fraction	帶分數
decimal point	小數點	fraction	分數	divisor	除數
units	個位	numerator	分子	dividend	被除數
tens	十位	denominator	分母	descending order	由大至小排列
tenths	十分位	proper fraction	真分數	ascending order	由小至大排列
hundredths	百分位	improper fraction	假分數	recurring decimals	循環小數

Useful Sentences

What fraction of the time is spent on studying?	花在學習上的時間佔幾分之幾？
What fraction of $\frac{7}{8}$ is $\frac{1}{2}$?	$\frac{1}{2}$ 是 $\frac{7}{8}$ 的幾分之幾？
Convert the following mixed fraction into an improper fraction.	把以下帶分數寫成假分數。
4.085 is read as four point zero eight five.	4.085 讀作四點零八五。
Which digit in 35.62 is in hundredths place?	在 35.62 這個數中，哪一個數字是在百分位？

Exercise 2

1. Consider the number 65.013. Determine whether each of the following is true for the number. Put a '✓' or a '×' in each of the boxes.

(a) '1' is in the hundredths place. ☐ **(b)** '1' in the number represents 10. ☐

(c) '6' in the number represents 60 000. ☐ **(d)** '0' is in the tenths place. ☐

(e) '5' in the number represents 50. ☐ **(f)** It is equal to $65\frac{13}{100}$. ☐

2. Reduce the following fractions into their simplest forms.

(a) $\frac{24}{80}$ _____ **(b)** $\frac{120}{135}$ _____ **(c)** $\frac{126}{42}$ _____

3. Convert the following decimals into fractions.

(a) 0.55 _____ (b) 3.75 _____ (c) 15.625 _____

4. Convert the following fractions into decimals.

(a) $\frac{98}{200}$ _____ (b) $3\frac{1}{8}$ _____ (c) $\frac{27}{25}$ _____

5. Compare the values of each of the following pairs of fractions. Put a '>' or '<' in each of the boxes.

(a) $\frac{4}{5} \square \frac{6}{7}$ (b) $\frac{11}{8} \square \frac{14}{11}$ (c) $\frac{5}{13} \square \frac{7}{17}$

6. Arrange each of the following sets of numbers in *ascending order*.

(a) 6.7, 8.03, 0.969, 10.34 _____

(b) 3.6, 3.06, 36, 30.6 _____

7. Arrange each of the following sets of numbers in descending order.

(a) $\frac{2}{3}, \frac{5}{6}, \frac{7}{12}, \frac{1}{2}$ _____

(b) 0.5, $\frac{1}{8}$, $1\frac{1}{16}$, 1.15, $1\frac{1}{4}$, 0.12 _____

Evaluate the following. (8 – 19)

8. $5.4-3.2+2.6$ _____

9. $19.8\div0.33$ _____

10. $12.6\div0.7-2.25\times4$ _____

11. $(6.6\times0.3-1.5)\times4$ _____

12. $\frac{11}{30}+\frac{5}{6}-\frac{7}{10}$ _____

13. $\frac{7}{30}\times\frac{6}{7}\times\frac{10}{12}$ _____

14. $1\frac{19}{45}\div32\div1\frac{1}{7}$ _____

15. $\left(3-\frac{2}{5}\right)\div2\frac{11}{35}$ _____

16. $\frac{7}{10}\div1\frac{1}{2}-\frac{1}{6}\times\frac{2}{3}$ _____

17. $\frac{2}{3}\times\left(\frac{11}{12}-\frac{5}{6}\right)+\frac{5}{8}$ _____

18. $2.5\div3\frac{3}{4}\times(2+0.75)$ _____

19. $0.25\times\left(2-\frac{7}{12}\right)\div1\frac{1}{16}$ _____

ascending order 由小至大排列

Find the result of each of the following. **(20 – 22)**

20. 6.5 L orange juice is divided into 26 cups equally. How much orange juice is there in each cup in L? _____

21. Wendy is 35 years old and Jacky is 15 years old. What fraction of Wendy's age is Jacky's age? _____

22. A bag of *coconuts* weighs 10.5 kg. If we sell 5 bags of coconuts for $756, how much does one kilogram of coconuts cost? _____

Solve the following problems. Show your working steps clearly. **(23 – 26)**

23. Each box of apples costs $35.8. It costs $4.3 more than each box of oranges. Nelson pays $250 for 5 boxes of oranges. How much change should he get?

24. **A bag of peanuts weighing** $2\frac{4}{5}$ **kg costs** $30. Mary buys a bag of peanuts that weighs $4\frac{2}{3}$ kg. **How much should she pay?**

coconut 椰子

25. Red roses cost \$50.4 per dozen, yellow roses cost \$81.6 per dozen. How much do 5 red roses and 8 yellow roses cost?

26. Linda uses $\frac{1}{7}$ of a bag of flour to make some bread and $\frac{3}{5}$ of the rest of it to make some biscuits. What fraction of the flour is left?