2022-2023 S4
2nd TERM UT
MATH EP
M1

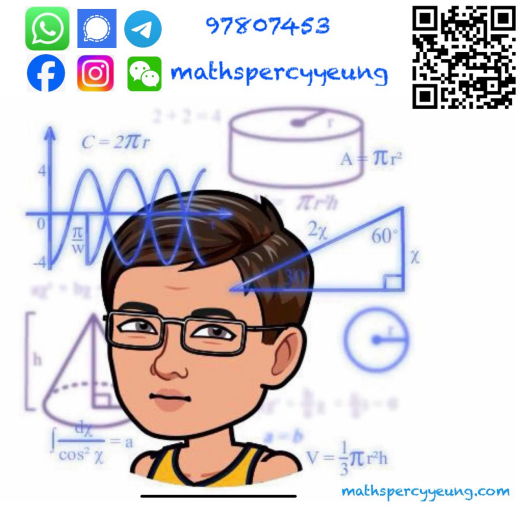
2022 – 2023
S4 Second Term Uniform Test

MATHEMATICS Extended Part

Module 1 (Calculus and Statistics)

Question–Answer Book

21st March, 2023
8:15 am – 9:15 am (1 hour)
This paper must be answered in English



Sections	Marks
A Total	**/28**
B Total	**/14**
TOTAL	**/42**

INSTRUCTIONS

1. Write your name, class and class number in the spaces provided on this cover.

2. This paper consists of TWO sections, A and B.

3. Attempt ALL questions in this paper. Write your answers in the spaces provided in this Question – Answer Book. Do not write in the margins. Answers written in the margins will not be marked.

4. Unless otherwise specified, all working must be clearly shown.

5. Unless otherwise specified, numerical answers should be either exact or given to 4 decimal places.

Section A (28 marks)

1. Find the equation of the tangent to the curve $y = \dfrac{3}{(5-2x)^2}$ at $x = 2$. (4 marks)

2. Differentiate the following functions with respect to x.

   (a) $y = xe^{-3x^2+9x-1}$

   (b) $y = \dfrac{\ln 5x}{e^{x^2}}$

(5 marks)

Answers written in the margins will not be marked

3. Let $f(x) = 4x^3 + mx^2 + nx + 615$, where m and n are constants. It is given that $(6, -33)$ is a turning point of the graph of $y = f(x)$.

(a) Find m and n.

(b) Find the local extrema of $f(x)$.

(6 marks)

Answers written in the margins will not be marked

4. A particle moves along a straight line. After t seconds, the displacement s (in cm) of the particle from a fixed point O is given by $s = 2kt^3 - kt^2 + k$, where k is a positive constant. Its initial displacement is 2 cm from O.

(a) Find the velocity of the particle at $t = 5$.

(b) Find the range of values of t that the acceleration of the particle is negative.

(6 marks)

Answers written in the margins will not be marked

5. If Winnie studies t minutes every day, she will score S marks in the test. The relation between S and t can be modelled by $S = 20 + \frac{2}{3}\sqrt{2t^2 - 60t + 2\ 025}$ where $15 \le t \le 90$. For the relation between her examination score P marks and S marks, it can be modelled by $P = -100 + 4S - 0.02S^2$.

(a) Find the rate of change of the test score of Winnie with respect to her time spent on studying when $t = 60$.

(b) Find the rate of change of Winnie's examination score with respect to her time spent on studying when $t = 60$.

(7 marks)

Answers written in the margins will not be marked

Section B (14 marks)

6. Let y be the oxygen content (in suitable units) in a fish pond. It is given that

$$y = (t^2 - 10t + h)e^{kt} + 16 \quad (0 \le t < 24),$$

where h and k are constants and t is the time measured in hour from 12:00 am of the day. The oxygen content is the lowest when $t = 3$. It is found that the initial oxygen content in the fish pond is 17 (in suitable units).

(a) Find h and k. (3 marks)

(b) Find the greatest oxygen content in the pond within that day. (4 marks)

(c) On that day, the fish is in danger when the oxygen content is 16 or below.

(i) How long is the fish in danger?

(ii) Find $\frac{d^2y}{dt^2}$.

(iii) Describe how $\frac{dy}{dt}$ varies within the period when the fish is in danger. Explain your answer.

(7 marks)

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END OF PAPER