2022-2023 S4
1st TERM EXAM
MATH EP
M1

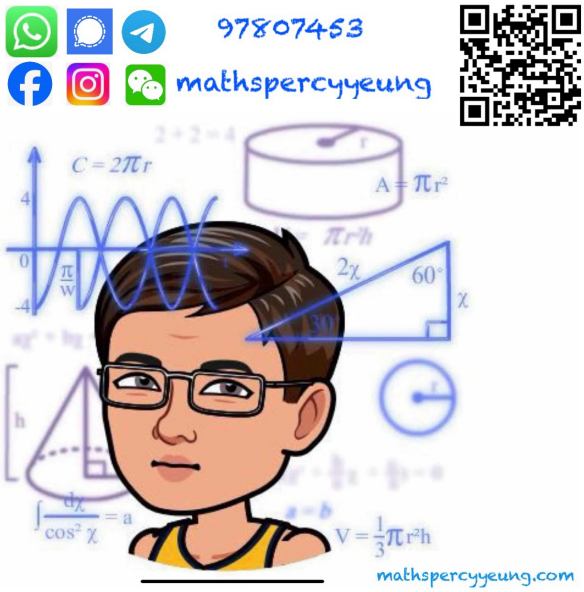
2022 – 2023
S4 First Term Examination

MATHEMATICS Extended Part

Module 1 (Calculus and Statistics)

Question–Answer Book

10th January, 2023
8:15 am – 9:30 am (1 hour 15 minutes)
This paper must be answered in English



Sections	Marks
A Total	**/30**
B Total	**/20**
TOTAL	**/50**

INSTRUCTIONS

1. Write your name, class and class number in the spaces provided on this cover.

2. This paper consists of TWO sections, A and B.

3. Attempt ALL questions in this paper. Write your answers in the spaces provided in this Question – Answer Book. Do not write in the margins. Answers written in the margins will not be marked.

4. Unless otherwise specified, all working must be clearly shown.

5. Unless otherwise specified, numerical answers should be either exact or given to 4 decimal places.

Section A (30 marks)

1. Find the coefficient of x^{93} in the expansion of $\left(\frac{1}{x}-x\right)^{99}$. (3 marks)

2. Given a curve C: $y=\frac{x^3}{3}+\frac{x^2}{2}-6x+3$.

   (a) Find the slope of the tangent to the curve C at the point $P(3,-\frac{3}{2})$.

   (b) Q is another point on the curve C. If the slopes of the tangents to the curve at P and Q are equal, find the coordinates of Q.

   (5 marks)

Answers written in the margins will not be marked

3. Evaluate the following limits.

(a) $\lim_{x \to 2} \frac{x^2 - x - 2}{x^2 - 3x + 2}$.

(b) $\lim_{x \to 9} \frac{\sqrt{x-5} - 2}{x^2 - 8x - 9}$.

(c) $\lim_{x \to +\infty} \frac{x^3 - 5x^2 + 11x - 12}{2x^3 + 14x^2 - 7}$.

(8 marks)

Answers written in the margins will not be marked

4. (a) Expand $(3x+2)^6$ in ascending powers of x up to the term x^2.

(b) Expand e^{-ax} in ascending powers of x up to the term x^2.

(c) If the coefficient of x^2 in the expansion of $\frac{(3x+2)^6}{e^{ax}}$ is 720, find the values of a.

(5 marks)

Answers written in the margins will not be marked

5. Find $\frac{dy}{dx}$ if

(a) $y = (7x - 12)(3x + 5)$,

(b) $y = \frac{x^2}{5x - 9}$.

(6 marks)

6. Let $f(x) = \frac{x}{\sqrt{x^2 - 4}}$. Find $f'(3)$. (3 marks)

Answers written in the margins will not be marked

Section B (20 marks)

7. In an experiment, the temperature (in °C) of a certain liquid can be modelled by

$$S = \frac{200}{1+(2^{bt})a},$$

where a and b are constants and t is the number of hours elapsed since the start of the experiment.

(a) Express $\ln\left(\frac{200}{S}-1\right)$ as a linear function of t. (2 marks)

(b) It is found that the intercepts on the vertical axis and the horizontal axis of the graph of the linear function obtained in (a) are $\ln 4$ and 4 respectively.

(i) Find a and b.

(ii) Find $\frac{dS}{dt}$ and $\frac{d^2S}{dt^2}$.

(6 marks)

Answers written in the margins will not be marked

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8. Let $f(x)=\dfrac{(\ln x)^2}{2x}$.

(a) Find $f'(x)$. (2 marks)

(b) If $f'(k)=0$, where k is a constant, find the possible values of k. (2 marks)

Answers written in the margins will not be marked

9. The temperature T (in °C) of a bottle of juice taken out from a refrigerator after t minutes is given by $T = 25 - 21e^{-0.06t}$.

(a) Find the initial temperature of the bottle of juice. (2 marks)

(b) At least how long will it take for the temperature of the bottle of juice to be higher than 18°C? (Give your answer correct to the nearest minute.) (3 marks)

(c) Will the temperature of the bottle of juice reach 30°C? Justify your answer. (3 marks)

END OF PAPER

Answers written in the margins will not be marked