2021-2022 S4
2nd TERM EXAM
MATH EP
M1

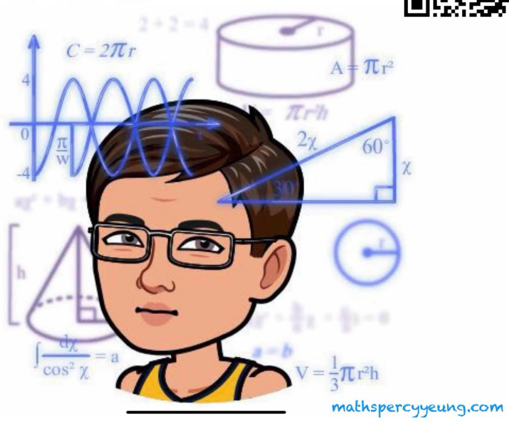
2021 – 2022
S4 Second Term Examination

MATHEMATICS Extended Part

Module 1 (Calculus and Statistics)

Question–Answer Book

22nd June, 2022
8:15 am – 9:45 am (1 hour 30 minutes)
This paper must be answered in English



Sections	Marks
A Total	**/43**
B Total	**/17**
TOTAL	**/60**

INSTRUCTIONS

1. Write your name, class and class number in the spaces provided on this cover.

2. This paper consists of TWO sections, A and B.

3. Attempt ALL questions in this paper. Write your answers in the spaces provided in this Question – Answer Book. Do not write in the margins. Answers written in the margins will not be marked.

4. Unless otherwise specified, all working must be clearly shown.

5. Unless otherwise specified, numerical answers should be either exact or given to 4 decimal places.

SECTION A (43 marks)

1. (a) Expand $(1+2x)^6$ in ascending powers of x up to the term x^2.

   (b) Find the coefficient of x^2 in the expansion of $e^{-x}(1+2x)^6$.

(5 marks)

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2. Evaluate $\lim_{x \to 1} \frac{x^3 - 1}{x^3 + x^2 - x - 1}$. (3 marks)

3. Solve the equation $\ln(2x+1) + \ln(x-6) = \ln 7$. (3 marks)

4. Find the global extrema of the function $f(x) = e^{-x^2+5x-3}$ for $1 \le x \le 4$. (5 marks)

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5. Differentiate the following functions with respect to x.

(a) $f(x)=(2x+1)^{\frac{3}{2}}(2x-1)^{\frac{3}{2}}$

(b) $g(x)=7^{x^3+x-1}$

(5 marks)

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6. Evaluate the following indefinite integral.

(a) $\int (2x-3)(x^2-3x+4)^8\,dx$

(b) $\int (6x-2)(x+1)\,e^{x^3+x^2-x}\,dx$

(6 marks)

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7. The slope of a curve at any point (x, y) is given by $\frac{dy}{dx} = 2x^2 - 4x + 7$. Given that $(3, 6)$ is a point lying on the curve.

(a) Find the equation of the curve.

(b) Find the equation of the tangent to the curve at $x = 1$.

(6 marks)

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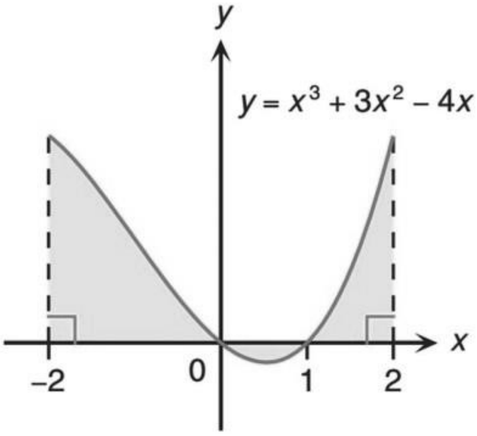
8. Find the area of the shaded region bounded by the curve $y = x^3 + 3x^2 - 4x$ and the x-axis from $x = -2$ to $x = 2$. (4 marks)



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9. Evaluate the following definite integrals:

(a) $\int_0^1 \frac{6x+9}{x^2+3x+4}\,dx$

(b) $\int_0^1 \frac{x^3+3x^2-2x-9}{x^2+3x+4}\,dx$

[Note: For definite integrals, answers obtained by using numerical integration functions in calculators are not accepted]

(6 marks)

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SECTION B (17 marks)

10. The rate of change of the number of participants $P(t)$ (in thousand) in an activity can be modelled by

$$P'(t) = A(t+4)e^{-kt}$$

where t ($t \geq 0$) is the number of days elapsed since the start of the activity, A and k are non-zero real constants. The following table shows some values of t and $P'(t)$.

t	1	2	3	4	5
$P'(t)$	9.27	8.23	7.11	6.03	5.01

(a) Express $\ln \dfrac{P'(t)}{t+4}$ as a linear function of t. (1 mark)

(b) Using the graph paper on Page 11, estimate the values of A and k correct to 1 decimal place. (5 marks)

(c) Find $P''(t)$. Hence prove that the rate of change of $P(t)$ is decreasing for $t \geq 0$. (4 marks)

(d) (i) Find $\dfrac{d}{dt}(t+4)e^{-nt}$, where n is a non-zero constant.

(ii) Hence, show that $\int (t+4)e^{-nt}dt = -\dfrac{e^{-nt}}{n^2} - \dfrac{(t+4)e^{-nt}}{n} + C$, where C is a constant.

(iii) It is known that there are 1 000 participants at the beginning of the activity. The organizer of the activity claims that there will be more than 60 000 participants 15 days elapsed since the start of the activity. Do you agree? Explain your answer. (7 marks)

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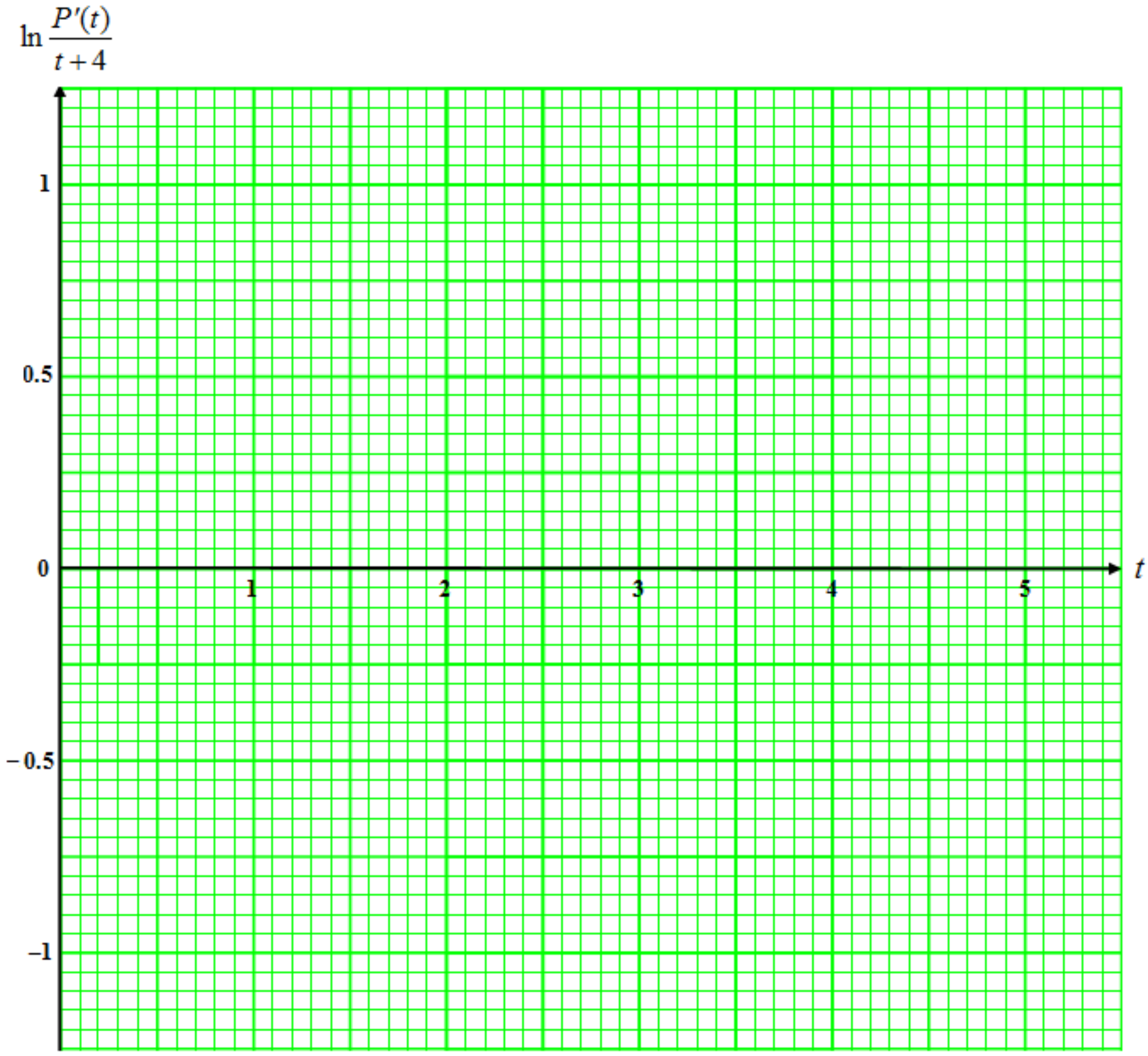

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