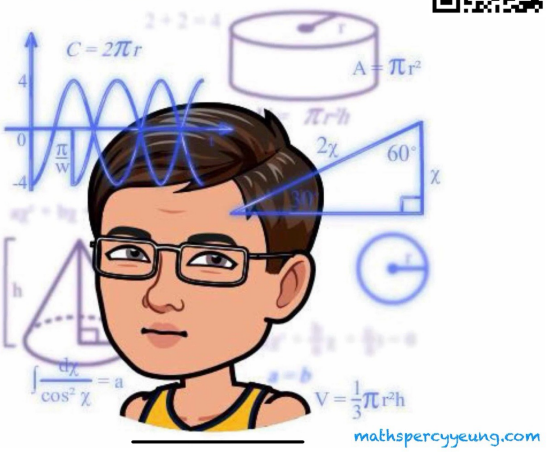

16-17 F.6
MOCK EXAM
MATH EP
M1

2016 – 2017
Form 6 Mock Examination

MATHEMATICS Extended Part

Module 1 (Calculus and Statistics)

Question–Answer Book

20 January, 2017
8:15 am – 10:45 am (2 hours 30 minutes)

This paper must be answered in English

Sections	Marks
A Total	**/50**
B Total	**/50**
TOTAL	**%**

INSTRUCTIONS

1. Write your name, class and class number in the spaces provided on this cover.

2. This paper consists of Section A and Section B.

3. Attempt ALL questions in this paper. Write your answers in the spaces provided in this Question – Answer Book. Do not write in the margins. Answers written in the margins will not be marked.

4. Unless otherwise specified, all working must be clearly shown. Numerical answers should be either exact or given to 4 decimal places.

5. The diagrams in this paper are not necessarily drawn to scale.

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Section A (50 marks)

1. Let X be a discrete random variable with probability function as shown in the following table.

x	-5	-2	0	k
$P(X = x)$	0.28	0.13	0.42	0.17

It is given that $\mathrm{E}(X) = -1.15$.

(a) Find the value of k.

(b) Find $\mathrm{Var}(2X + 5)$.

(c) Let A be the event that $X \leq -2$ and B be the event that $X > 2$. Find $P(A \cup B)$.

(5 marks)

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2. A and B are two events. Suppose that $\mathrm{P}(A) = 0.3, \mathrm{P}(B) = 0.28$ and $\mathrm{P}(B'|A') = 0.6$, where A' and B' are complementary events of A and B respectively.

(a) Find $\mathrm{P}(A' \cap B')$ and $\mathrm{P}(A' \cap B)$.

(b) Are A and B mutually exclusive? Explain your answer.

(6 marks)

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3. In a high jump competition, each team can select one athelete to participate. Each selected athelete has 3 attempts to jump. A team will win a medal if the athlete representing the team jumps higher than 2 m in at least two attempts. Team *A* consists of two atheletes, Benny and Sam. It is given that the probability that Benny is selected to participate the competition is 0.65, and the probabilities of Benny and Sam to jump higher than 2 m in an attempt are 0.6 and 0.45 respectively. Suppose their performances in each attempt are independent.

(a) Find the probability that team *A* will win a medal if Benny is selected.

(b) Find the probability that team *A* will win a medal.

(c) Given that team *A* cannot win a medal, find the probability that Benny is selected.

(7 marks)

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4. In a city, the population of teenagers x (in million) at time t (in years) can be modelled by

$$x = \frac{4(t + 0.2)}{\sqrt{t^2 + 2t + 2}}, \text{ where } t \geq 0.$$

On the other hand, the unemployment rate in the city R% can be modelled by

$$R = a\,e^{bx} \quad \text{where } a \text{ and } b \text{ are positive constants and } x \geq 0.$$

(a) Express ln R as a linear function of x.

(b) The values of R when t = 0, 1, 2 and 3 are recorded as follows:

t	0	1	2	3
R	2.13	4.01	5.17	5.89

(i) Using the graph paper on **Page 7**, estimate the values of a and b. Give your answers correct to 1 decimal place.

(ii) Using the estimates of a and b in (i), find the unemployment rate after a long period of time.

(7 marks)

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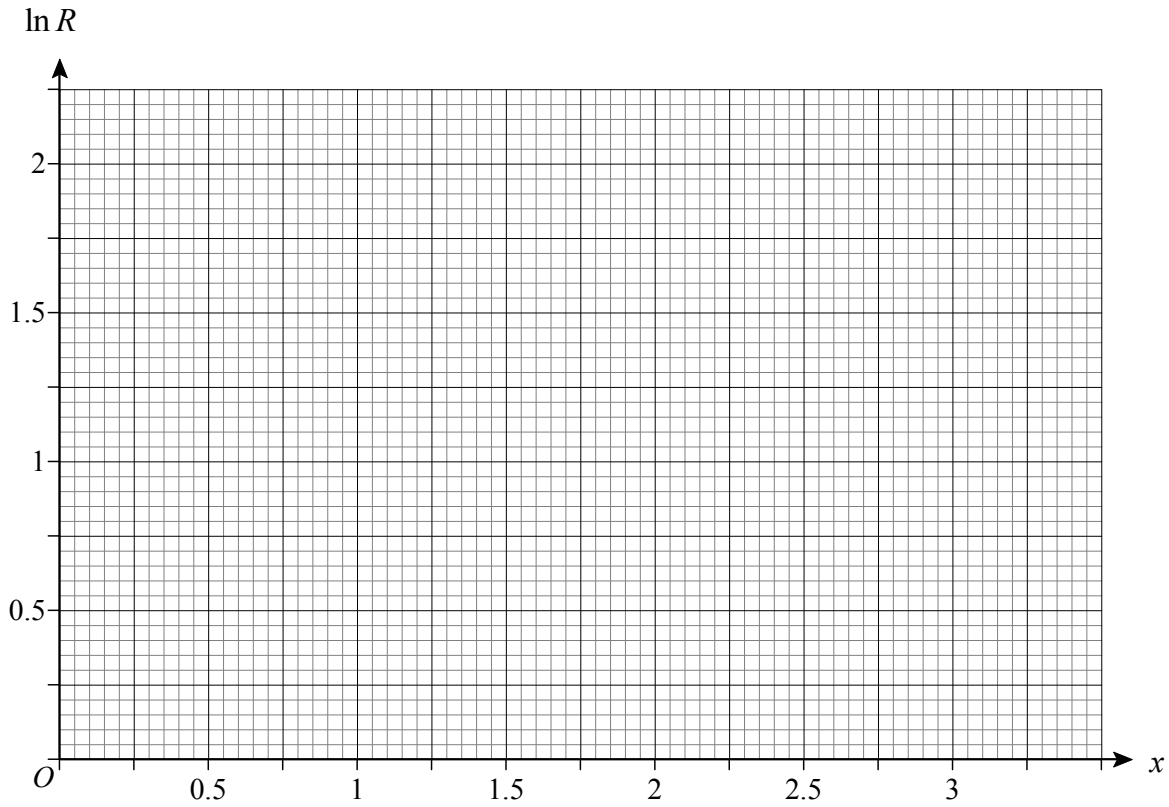

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5. The rate of change of the price P (in dollar) of an electronic device is given by $\frac{dP}{dt} = -t\,e^{-t^2+8}$, where t is the number of months since the beginning of 2015. The price of the electronic device at the beginning of 2015 is \$2000.

(a) Find P in terms of t.

(b) Find the price of the electronic device after a long time.

(6 marks)

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6. (a) Expand $(1-2a)^3$ in ascending powers of a.

(b) Express $e^{-2x}(1-2e^{kx})^3$ in ascending powers of x up to the term in x^2, where k is an integer.

(c) Suppose the coefficient of x^2 in the result of (b) is -101. Find the value of k.

(6 marks)

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7. Consider the curve $C : y = e^x + 1$. Let L be the tangent to C at the point $A(0,2)$ on C.

(a) Find the equation of L.

(b) Find the area bounded by C, L and the line $x = 2$.

[Note: For definite integrals, answers obtained by using numerical integration functions in calculators are not accepted.]

(6 marks)

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8. Define $f(x)=\dfrac{(\ln x)^2}{x}$ for all $x>0$. Let α and β be the two roots of the equation $f'(x)=0$, where $\alpha>\beta$.

(a) Express α in terms of e. Also find β.

(b) Using integration by substitution, evaluate $\int_{\beta}^{\alpha} f(x)\,dx$.

(7 marks)

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Section B (50 marks)

9. The cholesterol levels (in suitable units) of the adults in a city are assumed to be normally distributed with mean μ and variance σ^2. From a random sample of 49 adults, a 95% confidence interval for μ is found to be $(4.596, 5.044)$.

(a) (i) Find the value of σ.

(ii) Find the mean of the sample.

(3 marks)

(b) Another sample of 15 adults is randomly selected and their cholesterol levels are recorded as follows:

3.6	3.8	3.9	4.3	4.3	4.5	4.8	5.0
5.1	5.2	5.3	5.5	5.8	6.0	6.4	

The two samples are then combined. Construct a 99% confidence interval for μ using the combined sample.

(4 marks)

(c) A health organization classifies the cholesterol level of an adult to be low, medium and high if his/her cholesterol value is respectively at most 5.2, between 5.2 and 6.2, and at least 6.2. Suppose $\mu = 4.8$.

(i) Find the probability that the cholesterol level of a randomly selected adult in the city is low.

(ii) A sample of 20 adults is randomly selected in the city. Find the probability that there are more than 17 adults with low cholesterol level and at least 1 adult with medium cholesterol level in this sample.

(5 marks)

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10. In a factory, the number of flaws found in a carpet of $20\,m^2$ produced by machine A can be modelled by a Poisson distribution with a mean of 2.1. A carpet of $20\,m^2$ is regarded as *inferior* if more than 2 flaws are found in it. Machine A produces six carpets of 20 m^2 each day independently.

(a) Find the probability that a carpet of $20\,m^2$ produced by machine A is *inferior*.

(2 marks)

(b) If k consecutive days are observed, the probability of having at least one day with not more than 1 *inferior* carpet produced by machine A is greater than 0.99. Find the least value of k.

(4 marks)

(c) The factory owner buys a machine B to produce carpets of $20\,m^2$ together with machine A and the two machines operate independently. It is found that the number of flaws found in a carpet of $20\,m^2$ produced by machine B follows a Poisson distribution with a mean of 1.1. Machine B produces six carpets of $20\,m^2$ each day independently. A day is called a *lucky day* if not more than 2 *inferior* carpets are produced in total on that day.

(i) Find the probability that a day is a *lucky day*.

(ii) Given that a day is a *lucky day*, find the probability that all *inferior* carpets are produced by exactly one of the machines on that day.

(7 marks)

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11. Originally, there were 650 chickens on a farm. Then, avian influenza broke out. By the end of the 10th day of the outbreak, 253 chickens had died. A zoologist studies the case and models the number of chickens N on the farm by

$$N = \frac{Ae^{kt}}{t+10} \qquad (t \geq 0),$$

where A and k are constants and t is the number of days elapsed since the outbreak of the influenza.

(a) Find the values of A and k. (Give your answers correct to 2 decimal places if necessary.)

(2 marks)

(b) The government announces that if more than 50% of the chickens on a farm die, all the chickens on the farm must be killed.

(i) Using the values of A and k obtained in (a), find the rate of change of the number of chickens on the farm.

(ii) Will all the chickens be killed? Explain your answer.

(6 marks)

(c) The chicken farmer lost \$22 500 due to the avian influenza outbreak. He then decided to raise pigs instead of chickens. The rate of change of net profit \$$P$ from raising pigs can be modelled by

$$\frac{dP}{dt} = 125\left(e^{\frac{t}{20}} - e^{\frac{t}{40}}\right) \qquad (0 \leq t \leq 80),$$

where t is the number of days elapsed since the farmer begins raising pigs. Find the minimum number of days for the net profit from raising pigs to compensate for the loss from avian influenza

(4 marks)

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12. An investment consultant, Albert, predicts the total profit made by a factory in the coming year. He models the rate of change of profit (in million dollars per month) made by the factory by

$$A(t) = \ln\left(t^2 - 8t + 95\right),$$

where $t\,(0 \le t \le 12)$ is the number of months elapsed since the prediction begins. Let P_1 million dollars be the total profit made by the factory in the coming year under Albert's model.

(a) (i) Using the trapezoidal rule with 4 sub-intervals, estimate P_1.

(ii) Find $\dfrac{d^2A(t)}{dt^2}$.

(4 marks)

(b) The factory manager, Christine, models the rate of change of profit (in million dollars per month) made by the factory in the coming year by

$$B(t) = \frac{t+8}{\sqrt{t+3}},$$

where $t\,(0 \le t \le 12)$ is the number of months elapsed since the prediction begins. Let P_2 million dollars be the total profit made by the factory in the coming year under Christine's model.

(i) Find P_2.

(ii) Albert claims that the difference between P_1 and P_2 does not exceed 2. Do you agree? Explain your answer.

(9 marks)

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End of Paper

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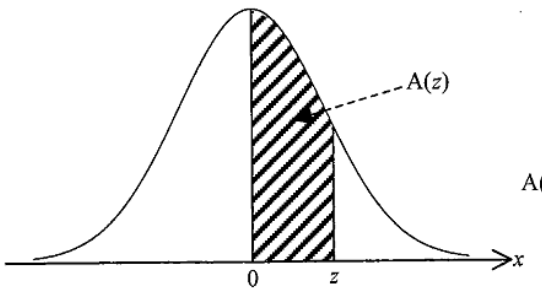
Standard Normal Distribution Table

z	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
0.0	.0000	.0040	.0080	.0120	.0160	.0199	.0239	.0279	.0319	.0359
0.1	.0398	.0438	.0478	.0517	.0557	.0596	.0636	.0675	.0714	.0753
0.2	.0793	.0832	.0871	.0910	.0948	.0987	.1026	.1064	.1103	.1141
0.3	.1179	.1217	.1255	.1293	.1331	.1368	.1406	.1443	.1480	.1517
0.4	.1554	.1591	.1628	.1664	.1700	.1736	.1772	.1808	.1844	.1879
0.5	.1915	.1950	.1985	.2019	.2054	.2088	.2123	.2157	.2190	.2224
0.6	.2257	.2291	.2324	.2357	.2389	.2422	.2454	.2486	.2517	.2549
0.7	.2580	.2611	.2642	.2673	.2704	.2734	.2764	.2794	.2823	.2852
0.8	.2881	.2910	.2939	.2967	.2995	.3023	.3051	.3078	.3106	.3133
0.9	.3159	.3186	.3212	.3238	.3264	.3289	.3315	.3340	.3365	.3389
1.0	.3413	.3438	.3461	.3485	.3508	.3531	.3554	.3577	.3599	.3621
1.1	.3643	.3665	.3686	.3708	.3729	.3749	.3770	.3790	.3810	.3830
1.2	.3849	.3869	.3888	.3907	.3925	.3944	.3962	.3980	.3997	.4015
1.3	.4032	.4049	.4066	.4082	.4099	.4115	.4131	.4147	.4162	.4177
1.4	.4192	.4207	.4222	.4236	.4251	.4265	.4279	.4292	.4306	.4319
1.5	.4332	.4345	.4357	.4370	.4382	.4394	.4406	.4418	.4429	.4441
1.6	.4452	.4463	.4474	.4484	.4495	.4505	.4515	.4525	.4535	.4545
1.7	.4554	.4564	.4573	.4582	.4591	.4599	.4608	.4616	.4625	.4633
1.8	.4641	.4649	.4656	.4664	.4671	.4678	.4686	.4693	.4699	.4706
1.9	.4713	.4719	.4726	.4732	.4738	.4744	.4750	.4756	.4761	.4767
2.0	.4772	.4778	.4783	.4788	.4793	.4798	.4803	.4808	.4812	.4817
2.1	.4821	.4826	.4830	.4834	.4838	.4842	.4846	.4850	.4854	.4857
2.2	.4861	.4864	.4868	.4871	.4875	.4878	.4881	.4884	.4887	.4890
2.3	.4893	.4896	.4898	.4901	.4904	.4906	.4909	.4911	.4913	.4916
2.4	.4918	.4920	.4922	.4925	.4927	.4929	.4931	.4932	.4934	.4936
2.5	.4938	.4940	.4941	.4943	.4945	.4946	.4948	.4949	.4951	.4952
2.6	.4953	.4955	.4956	.4957	.4959	.4960	.4961	.4962	.4963	.4964
2.7	.4965	.4966	.4967	.4968	.4969	.4970	.4971	.4972	.4973	.4974
2.8	.4974	.4975	.4976	.4977	.4977	.4978	.4979	.4979	.4980	.4981
2.9	.4981	.4982	.4982	.4983	.4984	.4984	.4985	.4985	.4986	.4986
3.0	.4987	.4987	.4987	.4988	.4988	.4989	.4989	.4989	.4990	.4990
3.1	.4990	.4991	.4991	.4991	.4992	.4992	.4992	.4992	.4993	.4993
3.2	.4993	.4993	.4994	.4994	.4994	.4994	.4994	.4995	.4995	.4995
3.3	.4995	.4995	.4995	.4996	.4996	.4996	.4996	.4996	.4996	.4997
3.4	.4997	.4997	.4997	.4997	.4997	.4997	.4997	.4997	.4997	.4998
3.5	.4998	.4998	.4998	.4998	.4998	.4998	.4998	.4998	.4998	.4998

Note : An entry in the table is the area under the standard normal curve between $x = 0$ and $x = z$ $(z \geq 0)$. Areas for negative values of z can be obtained by symmetry.



$$A(z) = \int_0^z \frac{1}{\sqrt{2\pi}} e^{\frac{-x^2}{2}} dx$$

Answers written in the margins will not be marked