**14 – 15 F.4**
**2nd Term Exam**
**MATH EP**
**M1**

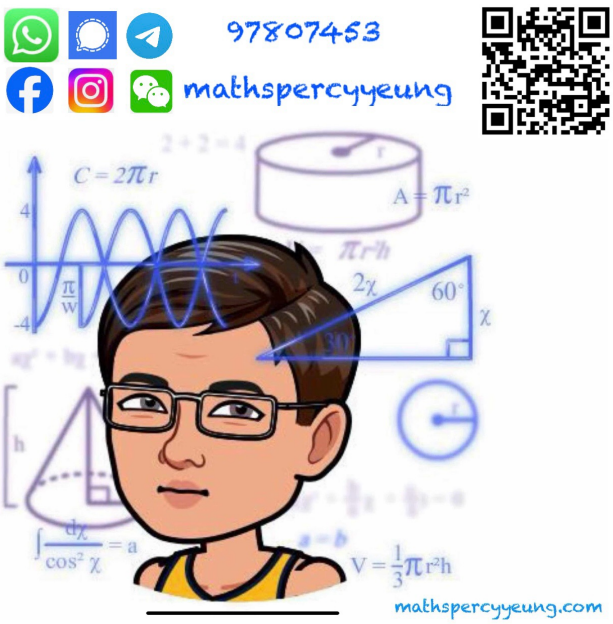
2014 – 2015
Form 4 Second Term Examination

# MATHEMATICS Extended Part

# Module 1 (Calculus and Statistics)

# Question–Answer Book

17th June, 2015
8:15 am – 9:45 am
Time Allowed: 1 hour 30 minutes
**This paper must be answered in English**




## INSTRUCTIONS

1. Write your name, class and class number in the spaces provided on this cover.

2. Answer **ALL** questions in this paper. Write your answers in the spaces provided in this Question–Answer Book. Do not write in the margins. Answers written in the margins will not be marked.

3. Unless otherwise specified, all working must be clearly shown.

4. Unless otherwise specified, numerical answers should be either exact or given to 4 decimal places.

5. The diagrams in this paper are not necessarily drawn to scale.

| | |
|---|---|
| **Section A Total** | **/37** |
| **Section B Total** | **/23** |
| **Grand Total** | **/60** |

**Section A (37 marks)**

1. (a) Expand $(1+3x)^8$ in ascending powers of $x$ up to the term $x^2$.

(b) Find the coefficient of $x^2$ in the expansion of $\frac{(1+3x)^8}{e^{2x}}$.

(4 marks)

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2. When a balloon is being deflated, its radius $r$ (in cm) will decrease with time $t$ (in s) which is given by $r = 5 + \frac{10}{1+t^2}$, where $t \geq 0$. It is known that the surface area $A$ (in $cm^2$) of the balloon is given by $A = 4\pi r^2$. Find the rate of change, in terms of π, of the surface area of the balloon when the radius is 10 cm.

(5 marks)

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3. Evaluate the following indefinite integral.

(a) $\int (x^2 - 3x + 1)\sqrt{x}\, dx$

(b) $\int (2x - 1)e^{5x^2 - 5x + 3}\, dx$

(6 marks)

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4. (a) Find $\frac{d}{dx}(x \ln x)$

(b) Using (a) to evaluate $\int_1^e \ln x \, dx$.

(4 marks)

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5. The slope of a curve $C$ at any point $(x, y)$ is given by $\frac{dy}{dx} = -3x + 2$. The straight line $y = 5x + 9$ is the tangent to the curve at point $A$.

(a) Find the coordinates of $A$.

(b) Find the equation of $C$.

(5 marks)

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6. Given that $f(x) = e^{-2x}(x^2 + 3x)$.

(a) Find $f'(x)$ and $f''(x)$.

(b) Hence solve the equation $f''(x) + f'(x) - 4f(x) = 3e^{-2x}$.

(5 marks)

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7. In a city, the population of elder people $x$ (in millions) at time $t$ (in years) can be modelled by

$$x = \frac{5(t+0.3)}{\sqrt{t^2+3t+6}} \quad \text{for} \quad t \geq 0.$$

On the other hand, the death rate of the city $R$% can be modelled by

$R = a\,e^{bx}$, where $a$ and $b$ are positive constants and $x \geq 0$.

(a) Express ln $R$ as a linear function of $x$.

(b) The values of $R$ when $t$ = 0, 1, 2 and 3 are recorded as follows:

| $t$ | 0 | 1 | 2 | 3 |
|---|---|---|---|---|
| $R$ | 1.28 | 1.47 | 1.60 | 1.68 |

(i) Using the graph paper on Page 9, estimate the values of $a$ and $b$. Correct your answers to 1 decimal place.

(ii) Using the estimate values of $a$ and $b$ in (i), find the death rate after a long period of time.

(8 marks)

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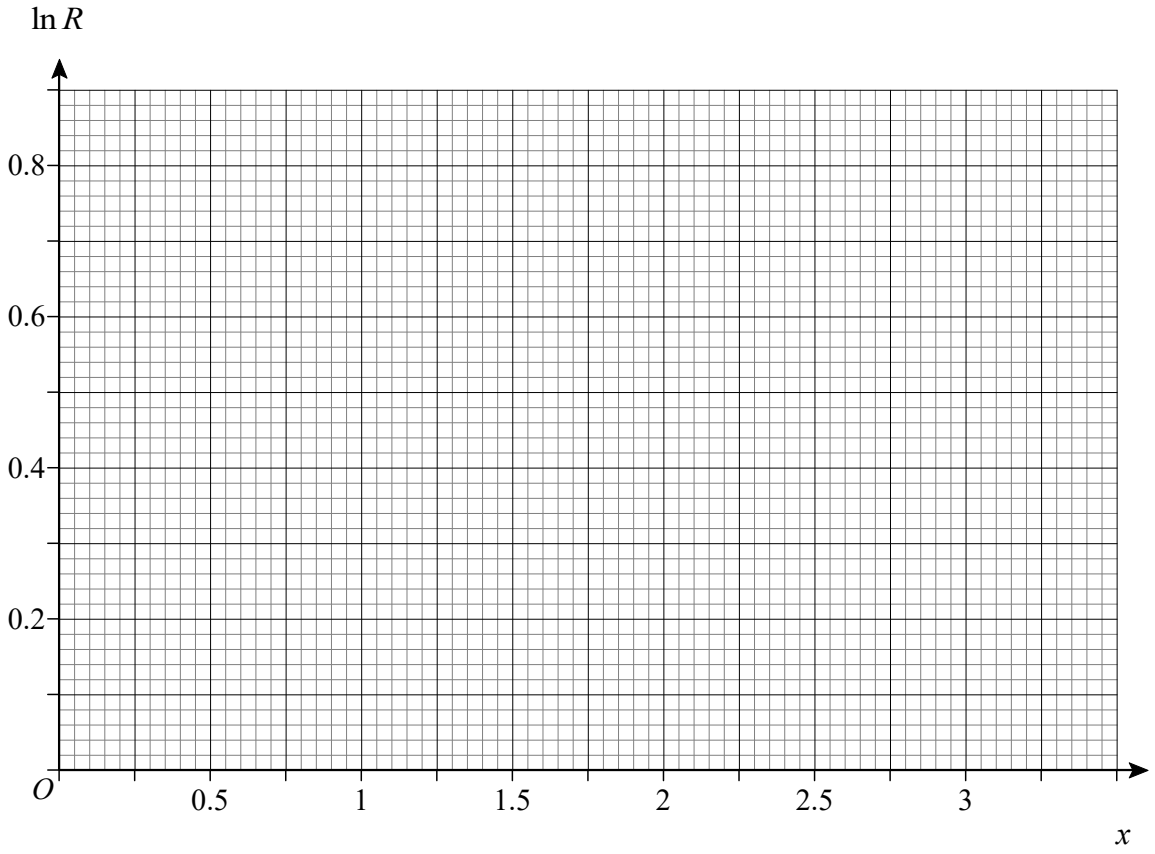


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**Section B (23 marks)**

8. A chemical is added to a bottle of liquid. The temperature $\theta(t)$ of the liquid can be modelled by

$$\theta(t) = \frac{4 + 20t + t^2}{e^{0.1t}} + k,$$

where $\theta(t)$ is measured in °C, $t$ ($t \geq 0$) is the time measured in seconds after the chemical has been added and $k$ is a constant. It is given that $\theta(0) = 14$.

(a) Find the value of $k$. (1 mark)

(b) Find $\frac{d\theta(t)}{dt}$. (3 marks)

(c) Will the temperature of the liquid get higher than 100°C? Explain your answer.

(6 marks)

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9. After starting the promotion of a movie, the rate of change of the daily number of visits to the official website of the movie can be modelled by

$$\frac{dN}{dt} = \frac{40k - kt}{e^{\frac{t}{40}} + 5t} \quad (t \geq 0) \ ,$$

where $N$ is the daily number of visits (in thousands) recorded at the end of a day, $t$ is the number of days elapsed since the start of the promotion and $k$ is a positive constant.

It is given that at the start of the promotion, $N = 2$ and $\frac{dN}{dt} = 4$ .

(a) (i) Let $u = 1 + 5te^{-\frac{t}{40}}$ , find $\frac{du}{dt}$ .

(ii) Find the value of $k$, and hence express $N$ in terms of $t$.

(7 marks)

(b) Find the greatest daily number of visits, correct to the nearest integer.

(2 marks)

(c) It is known that after 80 days from the start of the promotion, the rate of change of the daily number of visits becomes constant.

(i) Find the constant rate, correct to the nearest integer.

(ii) Find the daily number of visits after 100 days from the start of the promotion.

(4 marks)

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**End of Paper**

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