2020-2021
F4 FINAL EXAM
MATH EP
M2

School SY

F.4 FINAL EXAMINATION

2020 – 2021

MATHEMATICS Extended Part
Module 2 (Algebra and Calculus)

Question-Answer Book

11:00 am – 12:30 pm (1.5 hours)

This paper must be answered in English

Name	
Class	F.4 ( )
Class Number	

INSTRUCTIONS

1. After the announcement of the start of the examination, you should first write your Name, Class and Class Number in the space provided on Page 1.

2. Attempt ALL questions in this paper. Write your answers in the spaces provided in this Question-Answer Book. Do not write in the margins. Answers written in the margins will not be marked.

3. Graph paper and supplementary answer sheets will be supplied on request. Write your Name and mark the question number box on each sheet, and fasten them with string INSIDE the book.

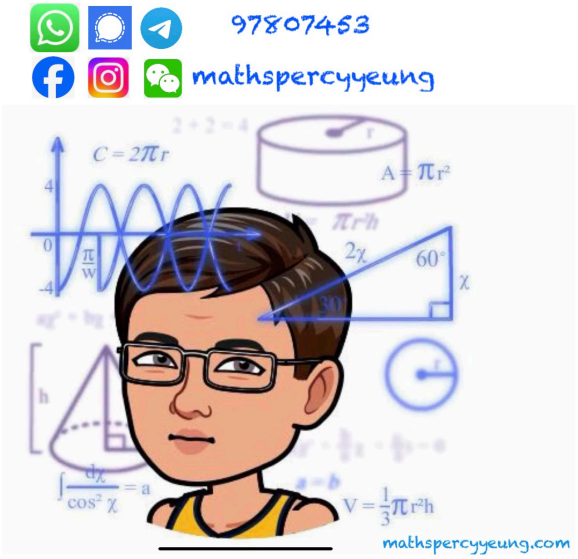
4. Unless otherwise specified, all working must be clearly shown.

5. Unless otherwise specified, numerical answers must be exact.

6. The diagrams in this paper are not necessarily drawn to scale.

7. No extra time will be given to candidates for writing names or filling in the question number boxes after the 'Time is up' announcement.

8. The full mark of this paper is 70.



FORMULAS FOR REFERENCE

$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$	$\sin A + \sin B = 2\sin\frac{A+B}{2}\cos\frac{A-B}{2}$
$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$	$\sin A - \sin B = 2\cos\frac{A+B}{2}\sin\frac{A-B}{2}$
$\tan(A \pm B) = \frac{\tan A \pm \tan B}{1 \mp \tan A \tan B}$	$\cos A + \cos B = 2\cos\frac{A+B}{2}\cos\frac{A-B}{2}$
$2\sin A \cos B = \sin(A+B) + \sin(A-B)$	$\cos A - \cos B = -2\sin\frac{A+B}{2}\sin\frac{A-B}{2}$
$2\cos A \cos B = \cos(A+B) + \cos(A-B)$	
$2\sin A \sin B = \cos(A-B) - \cos(A+B)$	

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1. Find and rationalize the exact value of $\tan\frac{\pi}{12}$.

(4 marks)

Answers written in the margins will not be marked

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2. Find the constant term in the expansion of $\left(x^2+\frac{3}{x}\right)^{15}$.

(5 marks)

3. Let $f(x)=\sqrt{x^2+kx+1}$ where k is a constant. If $f'(0)=2021$, find the value of k.

(4 marks)

Answers written in the margins will not be marked

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4. (a) Expand $(1-3x)^{10}$ up to the x^2 term.

(b) Let k be a positive constant. If the coefficient of x and x^2 in the expansion of $(1-3x)^{10}(k+x)^2$ are p and 26 respectively, find the value of p.

(6 marks)

Answers written in the margins will not be marked

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5. It is given that $x^2 + xy + e^{y-1} = 1$. Find $\left.\frac{dy}{dx}\right|_{x=0}$.

(5 marks)

6. Let $y = \frac{(\ln x)^2}{x}$. Show that $x^3 \frac{d^2y}{dx^2} + 3x^2 \frac{dy}{dx} + xy = 2$.

(4 marks)

Answers written in the margins will not be marked

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7. (a) Prove, by mathematical induction that,

$$\sum_{k=1}^{n} k(k+1)(k+2) = \frac{n(n+1)(n+2)(n+3)}{4} \quad \text{for all} \quad n \in \mathbb{N}.$$

(b) Hence, evaluate $\sum_{k=12}^{34} (k+1)(k+2)(k+3)$.

(8 marks)

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8. Let $f(x) = \sin^2 x$. Solve $f'(x) = f(2x) - 2$ for $0 \le x < \pi$.

(5 marks)

9. Let $f(x) = (x+2)^{\cos x}$. Find $f'(0)$.

(5 marks)

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10. (a) It is given that $\cos x \neq 0$. Show that $\cos 2x = \dfrac{1-\tan^2 x}{1+\tan^2 x}$.

(b) Hence, solve the equation $\dfrac{1-\tan^2 x}{1+\tan^2 x} - \dfrac{1-\cot^2 x}{1+\cot^2 x} = 1$ for $0 \leq x < 2\pi$.

(7 marks)

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11. (a) It is given that a, b and k are non-zero constants where $2\sin kx \sin\frac{x}{2} = \cos ax - \cos bx$.

Express a and b in terms of k.

(b) (i) Using (a), show that, for all $n \in \mathbb{N}$,

$$\sin x + \sin 2x + \sin 3x + \cdots + \sin nx = \frac{\sin\frac{n+1}{2}x \sin\frac{n}{2}x}{\sin\frac{x}{2}}.$$

(ii) Hence, show that $\sin\frac{\pi}{5} + \sin\frac{2\pi}{5} + \sin\frac{3\pi}{5} + \sin\frac{4\pi}{5} = \cot\frac{\pi}{10}$.

(10 marks)

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12. Let $f(x) = x(1+x)^n$, where n is a positive integer.

(a) Find the value of $f'(1)$ in terms of n.

(b) By considering the binomial expansion of $(1+x)^n$ and using the result of (a), evaluate $C_0^{20} + 2C_1^{20} + 3C_2^{20} + \cdots + 20C_{19}^{20} + 21C_{20}^{20}$.

(7 marks)

END OF PAPER

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