## 2022-2023-S5 2nd TERM UT-MATH-CP 1

2022-2023 S5 $2^{\text {nd }}$ TERM UT MATH CP PAPER 1

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2022-2023
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S5 Second Term Uniform Test

## MATHEMATICS Compulsory Part

## PAPER 1

## Question-Answer Book

$20^{\text {th }}$ March, 2023
8:15 am - 9:30 am (1 hour 15 minutes)
This paper must be answered in English

## INSTRUCTIONS

1. Write your name, class and class number in the spaces provided on this cover.
2. This paper consists of THREE sections, $\mathrm{A}(1)$, $\mathrm{A}(2)$ and B .
3. Attempt ALL questions in this paper. Write your answers in the spaces provided in this Question - Answer Book. Do not write in the margins. Answers written in the margins will not be marked.
4. Unless otherwise specified, all working must be clearly shown.
5. Unless otherwise specified, numerical answers should be either exact or correct to 3 significant figures.
6. The diagrams in this paper are not necessarily drawn to scale. daw


| Sections | Marks |
| :---: | :--- |
| $\mathrm{A}(1-4)$ |  |
| $\mathrm{A}(5-8)$ |  |
| A Total | $/ \mathbf{3 9}$ |
| B Total | $/ \mathbf{2 3}$ |
| TOTAL |  |

## Section A(1) (18 marks)

1. Simplify $\frac{\left(x^{-2} y^{3}\right)^{3}}{x^{-4} y}$ and express your answer with positive indices.
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2. Make $q$ the subject of the formula $p=\frac{5 q+3}{4 q}$.
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3. Factorize
(a) $16 m^{2}-9 n^{2}$,
(b) $16 m^{2}-9 n^{2}-8 m-6 n$.
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4. Jenny has several coins, 5 of them are ten-dollar coins, 2 of them are five-dollar coins, 1 of them is a two-dollar coin and 2 of them are one-dollar coins. If she selects a coin at random, find
(a) the probability of selecting a five-dollar coin,
(b) the expected value of the amount obtained.
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5. The coordinates of the points $M$ and $N$ are $(-1,2)$ and $(-4,-5)$ respectively. $M^{\prime}$ is the reflection image of $M$ with respect to the $x$-axis. $N$ is rotated anticlockwise about the origin $O$ through $270^{\circ}$ to $N^{\prime}$.
(a) Write down the coordinates of $M^{\prime}$ and $N^{\prime}$.
(b) Let $P$ be a moving point in the rectangular coordinate plane such that $P$ is equidistant from $M^{\prime}$ and $N^{\prime}$.
(i) Describe the geometric relation between the locus of $P$ and $M^{\prime} N^{\prime}$.
(ii) Find the equation of the locus of $P$.

## Section (2) (21 marks)

6. The stem-and-leaf diagram below shows the distribution of bonuses (in hundred dollars) of 20 salesmen of a company. It is known that the mean is equal to the median of the distribution.

## Distribution of bonuses of $\mathbf{2 0}$ salesmen of a company

Stem (thousands) Leaf (hundreds)

| 2 | 1 | 2 | 2 | 4 | 4 | 4 | 5 |  |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| 3 | 0 | 2 | $x$ | 3 | 3 | 5 | 6 | 8 |
| 4 | $y$ | 3 | 8 | 8 | 9 |  |  |  |

(a) Find $x$ and $y$.
(b) (i) Amy is one of the salesmen. If her bonus gained is $\$ 4300$, find her standard score.
(ii) If the data which are equal to the mean are removed, find the percentage change of the standard score of Amy.
(Give your answers correct to 3 significant figures if necessary)
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7. In the figure, two straight lines $L_{1}$ and $L_{2}$ intersect at $A$. The equations of $L_{1}$ and $L_{2}$ are $2 x-3 y+18=0$ and $x+3 y-9=0$ respectively.

(a) Write down the system of inequalities whose solutions are represented by the shaded region.
(2 marks)
(b) Find the maximum value of $P=-3 x+5 y$ subject to the constraints obtained in (a).
(4 marks)

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8. A circle passes through $A(2,0)$ and $B(-3,-1)$. The centre $C$ of the circle lies on the $y$-axis.
(a) Find the coordinates of $C$.
(2 marks)
(b) Find the equation of the circle.
(c) Given that $P(-2, a)$ lies on the circle, find the value(s) of $a$.
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## Section B (23 marks)

9. 13 different teams from zone $A$, 9 different teams from zone $B$ and 10 different teams from zone $C$ join a chess competition. In how many ways can 4 teams in the semi-finals be formed if
(a) all teams come from the same zone?
(2 marks)
(b) at least 1 team from each zone?
(2 marks)
(c) exactly 2 teams from one zone and 2 teams from another zone?
10. $L$ is a straight line with slope $m$, where $m<0$, and passes through $P(0,5) . L$ touches the circle $C_{1}: x^{2}+y^{2}+6 x-8 y+17=0$ at $Q$ and $L$ cuts the $x$-axis at $R$. Let $A$ be the centre of the circle.
(a) Find the coordinates of $A$ and the radius of $C_{1}$.
(Leave your answers in surd form if necessary.)
(b) Find the value of $m$.
(c) Find the coordinates of $Q$ and $R$.
(d) $C_{2}$ is another circle with centre $B$ touching $L$ such that $A, R$ and $B$ are collinear. The ratio of radius of $C_{1}$ to that of $C_{2}$ is $2: 1$. Find the equation of $C_{2}$.
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11. Robert is a university student and he is taking one course in Psychology and one course in Marketing.
(a) He estimates his probability of passing the Psychology course is 0.45 and that of passing the Marketing course is 0.55 . Furthermore, he estimates his probability of passing at least one course is 0.9 . Find the probability that he passes both Psychology and Marketing courses.
(b) After the mid-term tests, Robert reassesses his probability of passing the Psychology course is 0.75 . Meanwhile, his probability of passing at least one course is 0.8 and that of passing both courses is 0.2 . If his probability of passing the Marketing course is less than 0.5 , he will drop the Marketing course. Will he drop the Marketing course? Explain your answer.

## END OF PAPER

