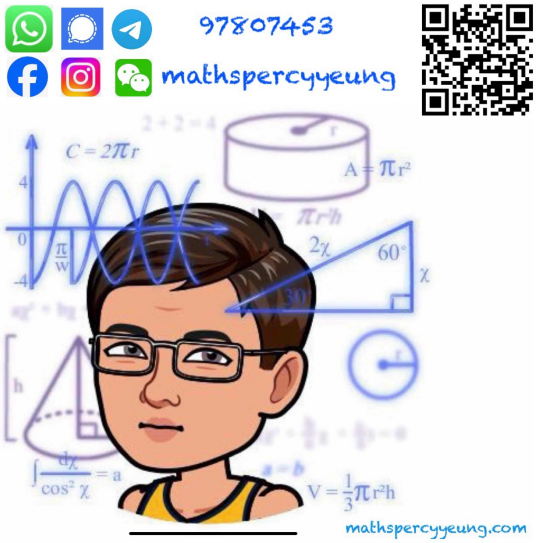
2022-2023 S5
2nd TERM EXAM
MATH CP
PAPER 1



2022 – 2023
S5 Second Term Examination

MATHEMATICS Compulsory Part

PAPER 1

Question–Answer Book

16th June, 2023
8:15 am – 10:30 am (2 hours 15 minutes)
This paper must be answered in English

INSTRUCTIONS

1. Write your name, class and class number in the spaces provided on this cover.

2. This paper consists of THREE sections, A(1), A(2) and B.

3. Attempt ALL questions in this paper. Write your answers in the spaces provided in this Question – Answer Book. Do not write in the margins. Answers written in the margins will not be marked.

4. Unless otherwise specified, all working must be clearly shown.

5. Unless otherwise specified, numerical answers should be either exact or correct to 3 significant figures.

6. The diagrams in this paper are not necessarily drawn to scale.

Sections	Marks
A (1 – 5)	
A (6 – 15)	
A Total	**/70**
B Total	**/35**
TOTAL	**/105**

Section A(1) (35 marks)

1. Simplify $\dfrac{x^{-2}y^{5}}{(x^{4}y^{-1})^{3}}$ and express your answer with positive indices. (3 marks)

2. Make y the subject of the formula $x(1-y)=\dfrac{y+3}{2}$. (3 marks)

3. Factorize

(a) $12x^{2}-xy-6y^{2}$,

(b) $6x+4y-12x^{2}+xy+6y^{2}$. (4 marks)

Answers written in the margins will not be marked

4. John buys some pencils at \$5 each and some pens at \$8 each. The total number of pencils and pens bought is 51. The amount spent on buying pencils is \$138 higher than that spent on buying pens. Find the amount of money spent on buying pens. (4 marks)

5. In a polar coordinate system, O is the pole. The polar coordinates of A and B are $(p, 45°)$ and $(p, 315°)$ respectively, where p is a positive constant. It is given that the distance between A and B is 26. Find

(a) $\angle AOB$,

(b) p,

(c) the area of ΔAOB.

(4 marks)

Answers written in the margins will not be marked

6. **(a)** Round up 429.251 to the nearest integer.

(b) Round down 429.251 to 1 decimal place.

(c) Round off 429.251 to 2 significant figures.

(3 marks)

7. Consider the compound inequality

$$5-2x \le \frac{4-x}{-2} \text{ and } 2x+1<19 \qquad \ldots\ldots\ldots\ldots\ldots(*).$$

(a) Solve (*).

(b) Find the number of integers satisfying the compound inequality in (a).

(4 marks)

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8. In Figure 1, DB is a diameter of the circle with centre O. $\angle DAC = 65°$ and $\angle ABD = 32°$. Find x and y. (4 marks)

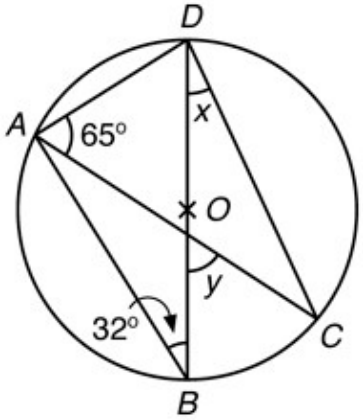

Figure 1

9. In Figure 2, $\angle POQ = 72°$ and the area of the sector POQ is 20π cm^2.

(a) Find the radius of the sector.

(b) Express the perimeter of the sector in terms of π.

(c) Find the area of the shaded region.

(6 marks)

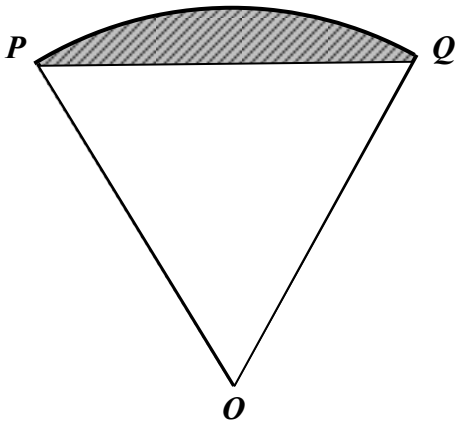

Figure 2

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Section A(2) (35 marks)

10. (a) Simplify $\dfrac{\cos(-\theta)\sin(180°-\theta)}{\tan(270°+\theta)}$.

(b) Hence, solve $\dfrac{\cos(-\theta)\sin(180°-\theta)}{\tan(270°+\theta)} = -\dfrac{1}{4}$, where $0° \le \theta \le 360°$.

(5 marks)

Answers written in the margins will not be marked

11. Let $f(x) = ax^3 - bx^2 - x + 2$ and $g(x) = x^3 + ax^2 - b$, where a and b are constants. $x - 1$ is a factor of $f(x)$. When $f(x)$ and $g(x)$ are divided by $x + 1$, the two remainders are equal.

(a) Find the values of a and b. (4 marks)

(b) Someone claims that all the roots of the equation $f(x) - 2g(x) = -1$ are irrational numbers. Do you agree? Explain your answer. (2 marks)

Answers written in the margins will not be marked

12. It is given that $f(x)$ is partly constant and partly varies as x^2. Suppose that $f(1) = -6$ and $f(-2) = -66$.

(a) Find $f(x)$. (3 marks)

(b) Consider the graph of $y = f(x) - 18x$. Find the axis of symmetry of the graph. (2 marks)

Answers written in the margins will not be marked

13. In Figure 3, *RS* is the altitude of the triangle *PQR* and it cuts the *y*-axis at *T*.

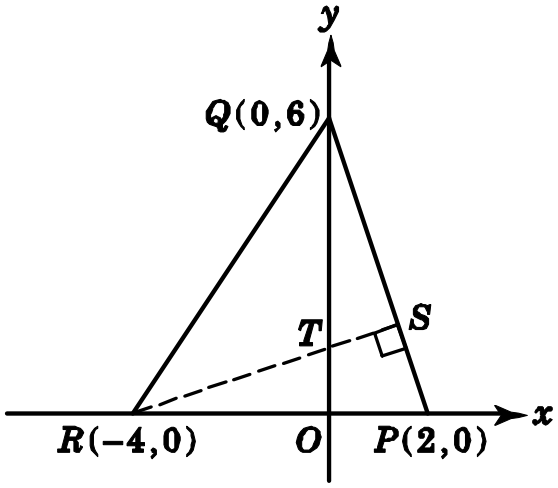


Figure 3

(a) Find the slopes of *PQ* and *RS*. (2 marks)

(b) Find the equation of *RS*. (2 marks)

Answers written in the margins will not be marked

14. The following stem-and-leaf diagram shows the numbers of working hours (per week) of 20 employees in a company.

The numbers of working hours (per week) of 20 employees in a company

Stem (Tens)	Leaf (Units)
1	4
2	
3	0 5 6 7 9
4	0 1 1 1 3 4 5 5 7 7 7 8 9
5	0

(a) Find the median, the range, the inter-quartile range and the standard deviation of the numbers of working hours (per week) of 20 employees in a company. (4 marks)

(b) Complete the box-and-whisker diagram in Figure 4(a) to show the distribution of the numbers of working hours (per week) of 20 employees in a company. (2 marks)

The numbers of working hours (per week) of 20 employees in a company

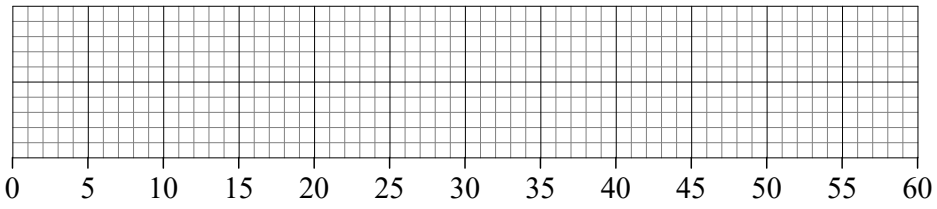
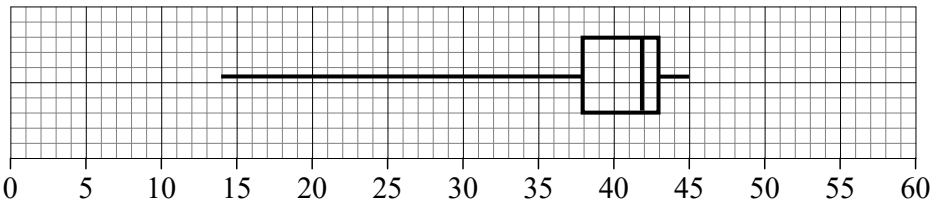


Figure 4(a)

Answers written in the margins will not be marked

14 (c) The boss of the company implements a new working scheme to shorten the working hours (per week) of each employee. The box-and-whisker diagram in Figure 4(b) shows the distribution of the numbers of new working hours (per week) of 20 employees in a company.

The numbers of working hours (per week) of 20 employees in a company



Numbers of working hours

Figure 4(b)

The boss claims that at least 25 % of the employees have their working hours shortened under the new scheme. Do you agree? Explain your answer. (2 marks)

Answers written in the margins will not be marked

15. The coordinates of the points A and B are (17, 3) and (7, 27) respectively. M is the mid-point of AB. Let P be a moving point in the rectangular coordinate plane such that $PM = AM$. Denote the locus of P by Γ.

(a) **(i)** Find the equation of Γ.

(ii) Describe the geometric relationship between AP and BP. (4 marks)

(b) Suppose that C is a point on Γ such that $AC = 10$, where the x-coordinate of C is greater than the x-coordinate of A. Find the area of ΔBCM. (3 marks)

Answers written in the margins will not be marked

Section B (35 marks)

16. In Figure 5, $ABCD$ is a quadrilateral garden. C is due east of A. The bearings of B and D from A are N37°E and S60°E respectively. The bearing of B from C is N50°W. $\angle ADC = 90°$ and $CD = 50$ m. Find the area of the garden. (5 marks)

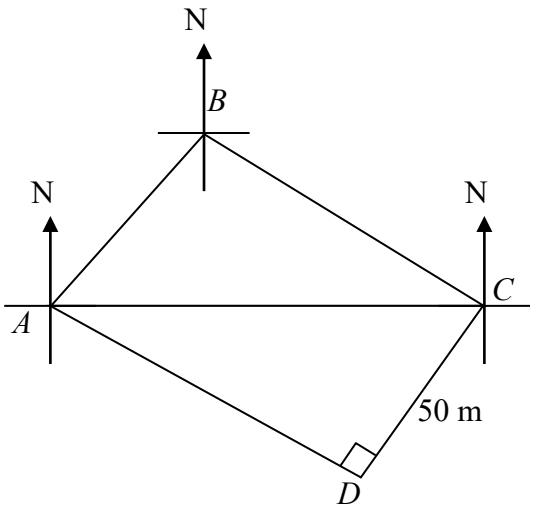


Figure 5

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17. In Figure 6, it shows the linear relation between $\log_4 y$ and x. The x-intercept and the intercept on the vertical axis of the graph are 10 and 5 respectively.

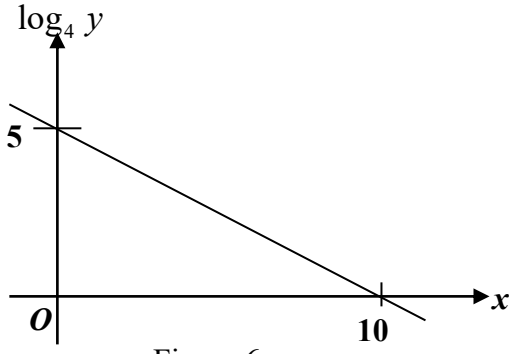

Figure 6

Express the relation between x and y in the form $y = Ak^x$, where A and k are constants.

(3 marks)

Answers written in the margins will not be marked

18. In Figure 7, the circle passes through four points A, B, E and C. MN is the tangent to the circle at A and is parallel to CB. AE and BC intersect at D, where D is the circumcentre of ΔABC. Show that ΔABC is a right-angled isosceles triangle. (4 marks)

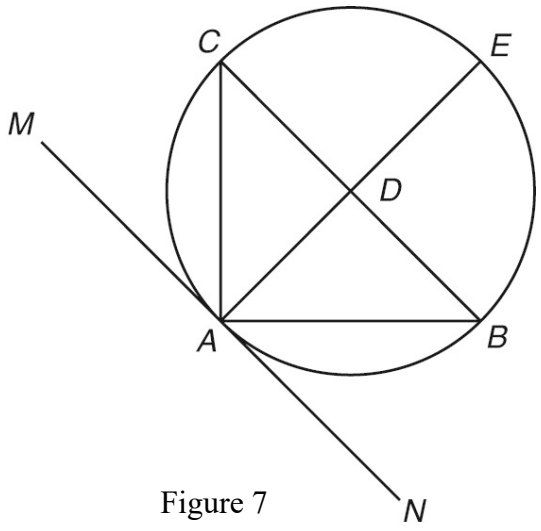

Figure 7

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19. Form a quadratic equation in x with roots $\frac{13}{2+3i}$ and $\frac{13}{2-3i}$. (4 marks)

Answers written in the margins will not be marked

20. It is given that $f(x) = -x^2 + 16x - 58$.

(a) Using the method of completing the square, express $f(x)$ in the form $a(x-h)^2 + k$, where a, h and k are real constants. (2 marks)

(b) The graph of $y = f(x)$ is reflected with respect to the x-axis and then translated upwards by 8 units to obtain the graph of $y = g(x)$. Find $g(x)$. (2 marks)

(c) Under a transformation, $f(x)$ is changed to $h(x) = -\frac{1}{4}x^2 + 8x - 58$.

(i) Describe the geometric meaning of the transformation.

(ii) Does the vertices of the graphs of $y = g(x)$ and $y = h(x)$ lie in the same quadrant? Explain your answer.

(4 marks)

Answers written in the margins will not be marked

21. In a fun fair, basketball shooting is a popular game. There are 6 baskets lying in a row and each basket can hold one ball only. A player has to shoot 3 balls. Assume that each ball is shot into a basket. If 3 balls are shot in alternate baskets, the player gets a grand prize(an example is shown in Figure 8(a)). If the balls are shot in 3 consecutive baskets, the player gets a big prize(an example is shown in Figure 8(b)).

Grand prize 

Figure 8(a)

Big prize 

Figure 8(b)

(a) Find the probability that a player gets a grand prize. (2 marks)

(b) David thinks that the probability of getting a big prize is twice that of getting a grand prize. Do you agree? Explain briefly. (3 marks)

(c) Are the events '3 balls are in alternate baskets' and 'no two balls are in consecutive baskets' equivalent? Explain your answer. (2 marks)

(d) If only 2 balls out of 3 are shot in consecutive baskets, a consolation prize will be given to the player. Peter uses the formula below to find the probability of getting a consolation prize.

$$P(\text{consolation prize}) = 1 - P(\text{grand prize}) - P(\text{big prize})$$

Is the formula correct? Explain your answer. If your answer is 'no', help Peter to find the required probability. (4 marks)

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