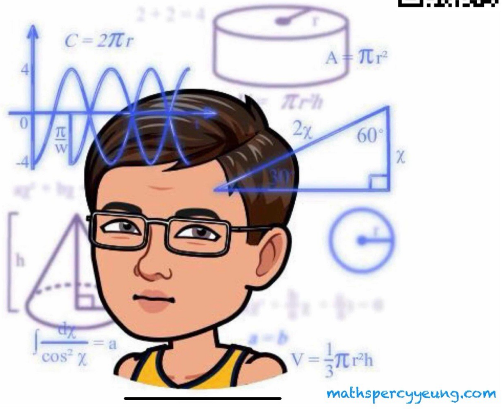
2022-2023 S5
1st TERM EXAM
MATH CP
PAPER 1



2022 – 2023
S5 First Term Examination

MATHEMATICS Compulsory Part

PAPER 1

Question–Answer Book

6th January, 2023
8:15 am – 10:00 am (1 hour 45 minutes)
This paper must be answered in English

INSTRUCTIONS

1. Write your name, class and class number in the spaces provided on this cover.

2. This paper consists of THREE sections, A(1), A(2) and B.

3. Attempt ALL questions in this paper. Write your answers in the spaces provided in this Question-Answer Book. Do not write in the margins. Answers written in the margins will not be marked.

4. Unless otherwise specified, all working must be clearly shown.

5. Unless otherwise specified, numerical answers should be either exact or correct to 3 significant figures.

6. The diagrams in this paper are not necessarily drawn to scale.

Section	Marks
A (1 – 3)	/ 10
A (4 – 11)	/ 44
A Total	**/ 54**
B Total	**/ 30**
TOTAL	**/ 84**

Answers written in the margins will not be marked

Section A(1) (28 marks)

1. Simplify $\dfrac{x\,y^{-5}}{x^{-9}\left(3y^{3}\right)^{2}}$ and express your answer with positive indices. (3 marks)

2. Factorize

(a) $9m^2 - 16$,

(b) $3mn + 4n + 9m^2 - 16$.

(3 marks)

3. Figure 1 shows a solid consisting of a hemisphere of radius r cm joined to the base of a right circular cylinder of height 27 cm and base radius r cm . It is given that the curved surface area of the circular cylinder is 972π cm^2 .

(a) Find r .

(b) Express the volume of the solid in terms of π .

(4 marks)

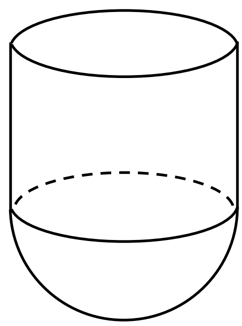


Figure 1

Answers written in the margins will not be marked

4. If the quadratic equation $x^2 + (k + 5)x + k^2 = 0$ has equal real roots, where k is a constant, find the values of k. (3 marks)

5. It is given that z varies directly as x^2 and inversely as y^3. Suppose that $z = 6$ when $x = 3$ and $y = 1$.

(a) Express z in terms of x and y.

(b) If $y = 2$ and $z = 3$, find x.

(5 marks)

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6. **(a)** **(i)** Solve the compound inequality $7(x-2) \le \frac{11x+8}{3}$ and $4-x<6$.

(ii) Solve the inequality $5x^2 + 11x < 12$.

(b) Find the values of x which satisfy

$$(\ 7(x-2) \le \frac{11x+8}{3} \text{ and } 4-x<6 \ ) \quad \text{or} \quad 5x^2 + 11x < 12 .$$

(6 marks)

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7. In Figure 2, *ABCDE* is a pentagon inscribed in a circle. $\angle ABC = 110°$. $\overset{\frown}{ABC} : \overset{\frown}{CD} = 5 : 1$. Find $\angle AED$. (4 marks)

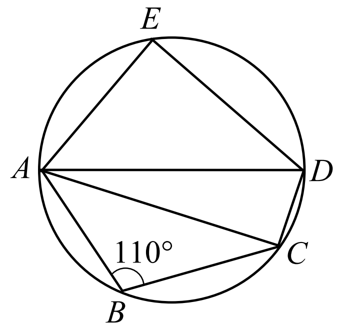

Figure 2

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Section A(2) (26 marks)

8. A factory introduces a new machine for manufacturing water pipes. At first, 20 trial pipes are produced. The factory manager records the lengths of the 20 trial pipes in the following stem-and-leaf diagram.

Stem (10 cm)	Leaf (1 cm)
12	a 7 8 8 8 8 9
13	0 5 6 6 7 8
14	0 1 b 4
15	3 8 9

It is given that the range and the inter-quartile range of the lengths of the 20 trial pipes are 35 cm and 13 cm respectively.

(a) Find the values of a and b. (3 marks)

(b) Two more trial pipes are added to the above sample. It is found that the mean is decreased by 1 cm and the range is increased by 1 cm. Find the lengths of each of these additional trial pipes. (3 marks)

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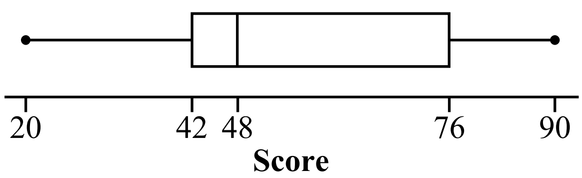
9. The box-and-whisker diagram shows the distribution of the scores of students in a test.



Ken and Billy are two of the students. The scores of Ken and Billy are 45 and 70 respectively. The standard scores of Ken and Billy are -1.2 and 0.8 respectively.

(a) Find the mean and the standard deviation of the scores of the students. (3 marks)

(b) Billy claims that the standard scores of less than half of the students are positive in the test. Do you agree? Explain your answer. (2 marks)

(c) If the datum 60 is removed from the scores of the students, will the standard score of Ken increase? Explain your answer. (2 marks)

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10. The circle C passes through the point $A(18, -24)$ and the centre of C is the point $G(8, 0)$.

(a) Find the equation of C. (2 marks)

(b) P is a moving point in the rectangular coordinate plane such that $AP = GP$. Denote the locus of P by L.

(i) Find the equation of L.

(ii) Describe the geometric relationship between L and the line segment AG.

(iii) If L cuts C at S and T, find the length of ST.

(5 marks)

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11. Let $p(x)$ be a cubic polynomial. When $p(x)$ is divided by $x-1$, the remainder is -5. When $p(x)$ is divided by $x+2$, the remainder is 91. It is given that $p(x)$ is divisible by $2x^2+8x-5$.

(a) Find the quotient when $p(x)$ is divided by $2x^2+8x-5$. (4 marks)

(b) Without using a calculator, find the sum of all the roots of the equation $p(x)=0$. (2 marks)

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Section B (30 marks)

12. If the real part and the imaginary part of $\dfrac{a-i}{3+4i}$ are equal, find the value of a. (3 marks)

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13. In Figure 3, O is the centre of a circle. From O, a straight line is drawn perpendicular to a line MN outside the circle and meets the line at point A. From A, another line is drawn to cut the circle at B and C respectively. The tangents at B and C cut the line MN at D and E respectively.

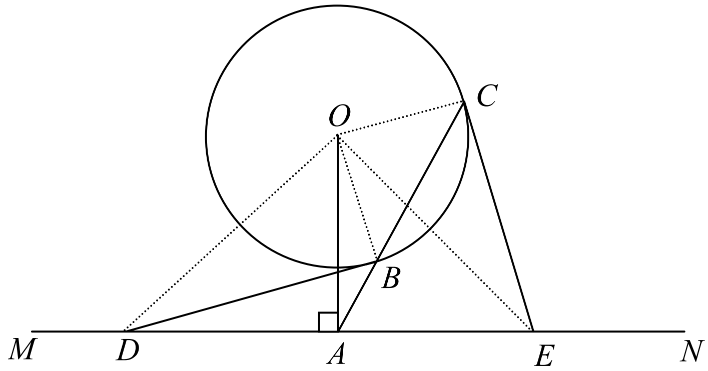

Figure 3

(a) **(i)** Prove that A, B, O, D are concyclic.

(ii) Prove that A, O, C, E are concyclic.

(4 marks)

(b) Hence, prove that $\triangle BDO \cong \triangle CEO$. (3 marks)

(c) Hence, or otherwise, prove that $AD = AE$. (3 marks)

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14. In Figure 4, L_1 passes through (0, 80) and (160, 0) . L_2 passes through (0, 200) and (80, 0) . The equation of L_3 is $y = \frac{3x}{2}$.

(a) Find the point of intersection of L_1 and L_2 . (3 marks)

(b) Isabella produces candy *X* and candy *Y*. 10 g of sugar and 50 g of jelly are required to produce a packet of candy *X*. 20 g of sugar and 20 g of jelly are required to produce a packet of candy *Y*. She has 1.6 kg of sugar and 4 kg of jelly. The number of packets of candy *Y* should be at most 1.5 times the number of packets of candy *X* . Suppose that *x* packets of candy *X* and *y* packets of candy *Y* are produced.

(i) Write down the constraints on x and y .

(ii) In Figure 4, shade the region representing the solutions of the system of inequalities in **(i)**.

(iii) If the profits from producing a packet of candy *X* and a packet of candy *Y* are \$ 5 and \$ 6 respectively, find the maximum profit.

(7 marks)

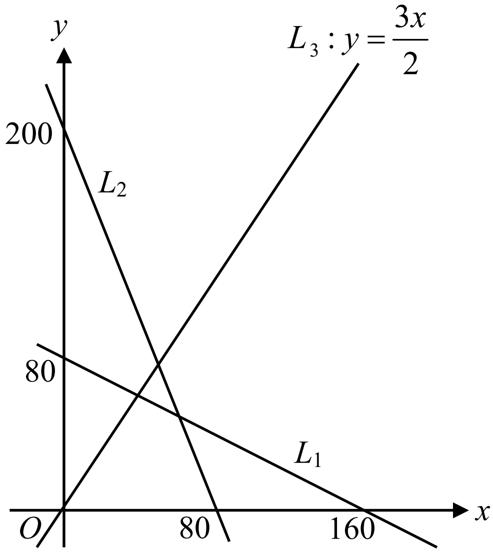

Figure 4

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15. A researcher models the number y of a kind of bacteria under controlled conditions by the formula $y = ab^x$, where x $(x \geq 0)$ is the number of days elapsed since the start of a research, and a and b are constants. The researcher plots a straight-line graph of $\log_2 y$ against x as shown in Figure 5.

(a) Find the values of a and b. (4 marks)

(b) If the number of bacteria on the t-th day is increased by 786 432 when compared with that on the previous day, find the value of t. (3 marks)

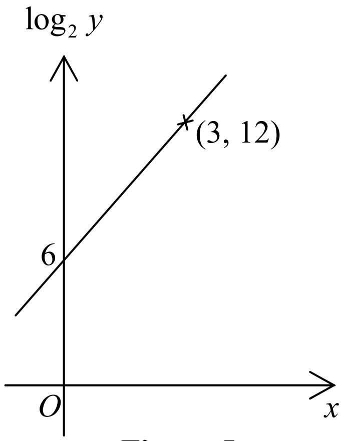

Figure 5

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