## 2021-2022 S5 2nd TERM UT-MATH-CP 1

2021-2022 S5
2nd TERM UT MATH CP PAPER 1

## MATHEMATICS Compulsory Part

## PAPER 1

## Question-Answer Book

$16^{\text {th }}$ May, 2022
8:15 am - 9:30 am (1 hour 15 minutes)
This paper must be answered in English

## INSTRUCTIONS

1. Write your name, class and class number in the spaces provided on this cover.
2. This paper consists of THREE sections, $\mathrm{A}(1)$, $\mathrm{A}(2)$ and B .
3. Attempt ALL questions in this paper. Write your answers in the spaces provided in this Question - Answer Book. Do not write in the margins. Answers written in the margins will not be marked.
4. Unless otherwise specified, all working must be clearly shown.
5. Unless otherwise specified, numerical answers should be either exact or correct to 3 significant figures.
6. The diagrams in this paper are not necessarily drawn to scale.


| Sections | Marks |
| :---: | :--- |
| $\mathrm{A}(1-5)$ |  |
| $\mathrm{A}(6-8)$ |  |
| A Total | $/ \mathbf{/ 4 0}$ |
| B Total | $/ \mathbf{6 0}$ |
| TOTAL |  |

## Section A(1) (20 marks)

1. Make $k$ the subject of the formula $\frac{a k+b}{k-b}=-3$.
2. Factorize
(a) $4 \alpha^{2}+12 \alpha+9$,
(b) $4 \alpha^{2}+12 \alpha+9-25 \beta^{2}$.
3. The marked price of a jacket is higher than its cost by $\$ 60$. The jacket is sold at a discount of $40 \%$ on its marked price. After selling the jacket, the percentage loss is $25 \%$. Find the marked price of the jacket.
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4. In Figure $1, D$ and $E$ are points on $A B$ and $C B$ respectively such that $A D=C E, B D=B E$ and $A E \perp B C$.
(a) Prove that $\triangle A B E \cong \triangle C B D$.
(b) It is given that $D C=12 \mathrm{~cm}$ and $C E=6 \mathrm{~cm}$.
(i) Is $\triangle B C D$ a right-angled triangle? Explain your answer.
(ii) Find the area of $\triangle B C D$.


Figure 1
(6 marks)
5. The coordinates of the points $A$ and $B$ are $(-5,2)$ and $(-3,4)$ respectively. $A^{\prime}$ is the reflection image of $A$ with respect to the $y$-axis. $B$ is rotated clockwise about the origin $O$ through $90^{\circ}$ to $B^{\prime}$.
(a) Write down the coordinates of $A^{\prime}$ and $B^{\prime}$.
(b) Are the lengths of $A B$ and $A^{\prime} B^{\prime}$ equal? Explain your answer.
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## Section A(2) (20 marks)

6. A manufacturer is going to produce two types of badminton rackets. Each badminton racket $A$ requires 10 units of aluminum and 2 units of carbon. Each badminton racket $B$ requires 3 units of aluminum and 9 units of carbon. There are at most 48 units of aluminum and 60 units of carbon for the production process. Let $x$ and $y$ be the numbers of badminton rackets $A$ and $B$ produced respectively.
(a) Write down all the constraints on $x$ and $y$.
(b) Using the graph paper on Page 5, represent the feasible solutions on a rectangular coordinate plane.
(c) If the profits of selling each badminton racket $A$ and each badminton racket $B$ are $\$ 40$ and $\$ 32$ respectively, find the maximum profit.

7. In the rectangular coordinate plane, the coordinates of points $A, B, C$ and $R$ are $(-1,0),(7,6)$, $(7,0)$ and $(3,3)$ respectively. $P$ is a moving point in the rectangular coordinate plane such that $A P$ is perpendicular to $B P$. Denote the locus of $P$ by $\Gamma$.
(a) (i) Find the equation of $\Gamma$.
(ii) Describe the geometric relationship between $\Gamma$ and $R$.
(3 marks)
(b) The equation of the straight line $L$ is $4 x+3 y+104=0$. It is found that $\Gamma$ and $L$ do not intersect. Let $Q$ be the nearest point on $L$ to $R$.
(i) Find the distance between $Q$ and $R$.
(ii) Wendy claims that the ratio of the area of $\triangle A R C$ to the area of $\triangle A Q C$ is 2:7. Do you agree? Explain your answer.
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8. The mean and the standard deviation of the test marks of a class of students are $\mu$ and $\sigma$ respectively. The test marks of Peggy and Sandy are 42 and 60 respectively. The standard scores of Peggy and Sandy are -0.5 and 1 respectively.
(a) Find the values of $\mu$ and $\sigma$.
(b) Later, the teacher found that the result of a student who got 48 marks was mistakenly not included in the calculation of $\mu$ and $\sigma$. How will the standard scores of Peggy and Sandy change when the missing test mark is included?
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## Section B (20 marks)

9. The equation of the circle $C$ is $x^{2}+y^{2}-16 x-4 y+52=0$. Denote the centre of $C$ by $G$.
(a) Find the coordinates of $G$ and the radius of the circle $C$.
(b) The straight line $L: y=x-k \quad$ cuts the circle $C$ at $P$ and $Q$. Denote the mid-point of $P$ and $Q$ by $M$.
(i) Express the coordinates of $M$ in terms of $k$.
(ii) If the length of the chord $P Q$ is $2 \sqrt{14}$ units, find $k$.
10. A box contains 4 black balls and 6 red balls. 5 balls are randomly drawn from the box at the same time.
(a) Find the probability that all the 5 balls drawn are red.
(b) Find the probability that the number of red balls drawn is more than the number of black balls drawn.
11. The following frequency distribution table shows the heights (in cm ) of plants $H$ in Farm $A$ and Farm B.

| Height $(h \mathrm{~cm})$ | Frequency |  |
| :---: | :---: | :---: |
|  | Farm $A$ | Farm $B$ |
| $15 \leq h<17$ | 17 | 13 |
| $17 \leq h<19$ | 45 | 15 |
| $19 \leq h<21$ | 66 | 17 |
| $21 \leq h<23$ | 14 | 33 |
| $23 \leq h<25$ | 8 | 37 |
| $25 \leq h<27$ | 10 | 45 |

When the height of a plant $H$ reaches 21 cm or above, it can be sold in the market.
(a) (i) If a plant $H$ is selected randomly from Farm $A$, find the probability that the selected plant $H$ can be sold in the market.
(ii) If two plants $H$ are selected randomly from Farm $A$, find the probability that at least one of the selected plants $H$ can be sold in the market.
(b) The following are two methods of selecting two plants $H$ from these two farms.

Method 1: Choose a farm randomly from these two farms, and then select two plants $H$ at random from the farm chosen.
Method 2: Select a plant H randomly from each of these two farms.

Which method has a greater chance of selecting at least one plant $H$ that can be sold in the market? Explain your answer.
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