## 2021-2022 S5 1st TERM EXAM-MATH-CP 1

2021-2022 S5
1st TERM EXAM
MATH CP
PAPER 1
MATHEMATICS Compulsory Part

## PAPER 1

## Question-Answer Book

$6^{\text {th }}$ January, 2022
8:15 am - 10:00 am (1 hour 45 minutes) This paper must be answered in English

## INSTRUCTIONS

1. Write your name, class and class number in the spaces provided on this cover.
2. This paper consists of THREE sections, $\mathrm{A}(1)$, $\mathrm{A}(2)$ and B .
3. Attempt ALL questions in this paper. Write your answers in the spaces provided in this Question - Answer Book. Do not write in the margins. Answers written in the margins will not be marked.
4. Unless otherwise specified, all working must be clearly shown.
5. Unless otherwise specified, numerical answers should be either exact or correct to 3 significant figures.
6. The diagrams in this paper are not necessarily drawn to scale.


| Sections | Marks |
| :---: | ---: |
| A $(1,2,10)$ |  |
| A (3-9) |  |
| A Total | $/ \mathbf{6 4}$ |
| B Total | $/ \mathbf{8 4}$ |
| TOTAL |  |

## Section A(1) (24 marks)

1. Simplify $\left(\frac{x^{2}}{2 y^{-1}}\right)^{-3}$ and express your answer with positive indices.
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2. In Figure 1, the area of the sector is $162 \pi \mathrm{~cm}^{2}$.
(a) Find the radius of the sector.
(b) Find the perimeter of the sector in terms of $\pi$.
(5 marks)


Figure 1
3. (a) Solve the inequality $3 x+6 \geq 4+x$.
(b) Find all integers which satisfy both the inequalities $3 x+6 \geq 4+x$ and $2 x-5<0$.
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Answers written in the margins will not be marked
4. The marks scored by eleven students in a Mathematics quiz are as follows:

| 10 | 60 | 45 | 20 | 30 | 70 | 70 | 65 | 65 | 50 | 65 |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |

Find the mean, mode, median, range and inter-quartile range of the above marks. (6 marks)
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5. In Figure 2, $A B=B C, \widehat{B C}=8 \mathrm{~cm}, \overparen{C D}=12 \mathrm{~cm}$. If $\angle D B C=45^{\circ}$, find $x$ and $y$. (6 marks)


Figure 2

## Section A(2) (40 marks)

6. The cumulative frequency polygon in Figure 3 shows the distribution of the yearly average scores of all the Secondary 2 students in School A.

The cumulative frequency polygon of the yearly average scores of all the Secondary 2 students in School A


Answers written in the margins will not be marked
(a) Find the median of the yearly average scores.
(b) The students will be allocated to 3 different groups in Secondary 3 according to their yearly average scores. The top $30 \%$ will be in Group I and the bottom $30 \%$ will be in Group III. The rest will be in Group II. Find
(i) the minimum yearly average score for students to be allocated to Group I;
(ii) the minimum yearly average score for students to be allocated to Group II.
(c) Complete Table 1.
(d) From Table 1, find the mean and standard deviation of the yearly average scores.

Table 1 The frequency distribution table of the yearly average scores of all Secondary 2 students in School A.

| Yearly average score $(x)$ | Class mark | Frequency |
| :---: | :---: | :---: |
| $20<x \leq 30$ | 25 | 12 |
| $30<x \leq 40$ |  | 20 |
| $40<x \leq 50$ |  | 32 |
|  |  | 30 |
| $60<x \leq 70$ |  |  |
| $80<x \leq 90$ |  |  |
|  | 95 |  |

7. The cost of a souvenir of surface area $A \mathrm{~cm}^{2}$ is $\$ C$. It is given that $C$ is the sum of two parts, one part varies directly as $A$ while the other part varies directly as $A^{2}$ and inversely as $n$, where $n$ is the number of souvenirs produced. When $A=50$ and $n=500, C=350$. When $A=20$ and $n=400, C=100$.
(a) Express $C$ in terms of $A$ and $n$.
(b) The selling price of a souvenir of surface area $A \mathrm{~cm}^{2}$ is $\$ 8 A$ and the profit in selling the souvenir is $\$ P$.
(i) Express $P$ in terms of $A$ and $n$.
(ii) Suppose $P: n=5: 32$. Find $A: n$.
(iii) Suppose $n=500$. Can a profit of $\$ 100$ be made in selling a souvenir? Explain your answer.
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Answers written in the margins will not be marked
8. In Figure 4, $A X$ is an altitude of $\triangle A B C$, and it cuts the $y$-axis at $P$.


Figure 4
(a) (i) Find the equation of $A X$.
(ii) Find the coordinates of $P$.
(b) Suppose $C Y$ is the altitude of $\triangle A B C$ on $A B$. Does $P$ lie on $C Y$ ? Explain your answer.
(c) Prove that the three altitudes of the $\triangle A B C$ pass through the same point.
9. Let $p(x)=6 x^{4}+7 x^{3}+a x^{2}+b x+c$, where $a, b$ and $c$ are constants. When $p(x)$ is divided by $x+2$ and when $p(x)$ is divided by $x-2$, the two remainders are equal. It is given that $p(x)=\left(l x^{2}+5 x+8\right)\left(2 x^{2}+m x+n\right)$, where $l, m$ and $n$ are constants.
(a) Find $l, m$ and $n$.
(b) Andy claims that the equation $p(x)=0$ has 4 real roots. Do you agree? Explain your answer.
(3 marks)
10. Figure 5 a shows an inverted cone with base radius 6 cm and height 8 cm . It contains water to a height of 4 cm .


Figure 5a


Figure 5b
(a) (i) Find the capacity of the cone, in terms of $\pi$.
(ii) Find the volume of water inside the cone, in terms of $\pi$.
(4 marks)
(b) The cone is then put upside down as shown in Figure 5 b and the height of the water level becomes $h \mathrm{~cm}$. Find $h$.
(4 marks)
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Answers written in the margins will not be marked

## Section B (20 marks)

11. In Figure 6, two circles $A B C$ and $A C D$ intersect at $A$ and $C$ respectively. $A B$ and $A D$ are the diameters of circles $A B C$ and $A C D$ respectively. $A D$ is the tangent to the circle $A B C$ at $A$.


Figure 6
(a) Prove that $B C D$ is a straight line.
(b) If $A B=12 \mathrm{~cm}$ and $A D=5 \mathrm{~cm}$, find the distance between the two centres of the circles.
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Answers written in the margins will not be marked
12. (a) The shaded region (including the boundary) in Figure 7 represents the solution of a system of inequalities. Find the system of inequalities.
(b) Let $P=2 x-y$. Find the greatest value of $P$ given $(x, y)$ is any point satisfying all the constraints in (a).
(2 marks)


Figure 7
13. The heights of the students in a school are normally distributed. The mean and the standard deviation of the heights are 168 cm and 4 cm respectively. If there are 192 students with heights less than 164 cm , find the number of students with heights between 160 cm and 176 cm .
(5 marks)

Answers written in the margins will not be marked
14. Given that $(2 x-i)(1+3 x i)=(y+2 i)-(8-y i)$. Find the values of $x$ and $y$ where $x$ and $y$ are integers.

