## 2020-2021 S4 2nd TERM EXAM-MATH-CP 1



MATHEMATICS Compulsory Part

## PAPER 1

## Question-Answer Book

$17^{\text {th }}$ June, 2021
8:15 am - 9:45 am (1 hour 30 minutes)
This paper must be answered in English

## INSTRUCTIONS

1. Write your name, class and class number in the spaces provided on this cover.
2. This paper consists of THREE sections, $\mathrm{A}(1)$, $\mathrm{A}(2)$ and B .
3. Attempt ALL questions in this paper. Write your answers in the spaces provided in this Question - Answer Book. Do not write in the margins. Answers written in the margins will not be marked.
4. Unless otherwise specified, all working must be clearly shown.
5. Unless otherwise specified, numerical answers should be either exact or correct to 3 significant figures.
6. The diagrams in this paper are not necessarily drawn to scale.

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| Sections | Marks |
| :---: | ---: |
| $\mathrm{A}(1-4)$ | $/ \mathbf{1 4}$ |
| $\mathrm{A}(5-9)$ | $/ 27$ |
| A Total | $/ \mathbf{1 9 1}$ |
| B Total | $/ 70$ |
| TOTAL |  |

## Section A(1) (18 marks)

1. Simplify $x^{-4}\left(\frac{x^{5}}{y^{-2}}\right)^{3}$ and express your answer with positive indices.
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2. Make $m$ the subject of the formula $\frac{6+n}{1-m}=4 n$. (3 marks)
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3. Factorize
(a) $a^{3}+a^{2} b-7 a^{2}$,
(b) $a^{3}+a^{2} b-7 a^{2}-a-b+7$.
4. The marked price of a birthday cake is $\$ 360$. The birthday cake is sold at a discount of $45 \%$ on its marked price. If the marked price of the birthday cake is $80 \%$ above its cost, find the percentage profit or percentage loss.
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5. It is given that the equation $15 x^{2}-6 x-4=6(x-k)$ has two distinct real roots.
(a) Find the range of values of $k$.
(b) If $k$ is an even number, find the greatest value of $k$.
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## Section A(2) (23 marks)

6. In Figure 1, $A, B, C$ and $D$ are points lying on the circle and $A B / / D C$. Find $x$ and $y$. (4 marks)

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Figure 1
7. It is given that $f(x)=8\left(10^{x}\right), g(x)=a\left(2^{x}\right)$ and $f(-1) \times g(3)=160$.
(a) Find the value of $a$.
(b) If $f(k)=g(2 k+1)$, find the value of $k$.
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8. In Figure 2, $C D$ is perpendicular to $A B$, and $C D$ cuts the $y$-axis at $E$.
(a) Find the equations of $A B$ and $C D$. (4 marks)
(b) Find the coordinates of $D$ and $E$.
(2 marks)
(c) Find the area of quadrilateral $O B D E$. (2 marks)


Figure 2
9. Let $f(x)=2 x^{3}-7 x^{2}+6 x-5$.
(a) Find the quotient and the remainder when $f(x)$ is divided by $x^{2}-4 x+3$. ( 3 marks)
(b) Let $g(x)=2 f(x)-(r x+s)$, where $r$ and $s$ are constants. It is given that $g(x)$ is divisible by $x^{2}-4 x+3$.
(i) Find the values of $r$ and $s$.
(ii) Hence, factorize $g(x)$ completely.

## Section B (29 marks)

10. A researcher investigated a certain species of frogs for a year. The population of the frogs $(y)$ after $x$ months can be estimated by $y=k(10)^{a x}$, where $a$ and $k$ are constants.
(a) Express $\log y$ in terms of $a, k$ and $x$.
(b) It is given that the graph of $\log y$ against $x$ in (a) is a straight line passing through the points $(0,3.2)$ and $(10,2.2)$. Find the values of $a$ and $k$.
(c) Using the results of (b), find the number of frogs remained at the end of the investigation.
11. Let $f(x)=x^{2}-2 p x+43$, where $p$ is a positive constant. It is given that the $y$-coordinate of the vertex of the graph of $y=f(x)$ is 27 .
(a) Find the value of $p$.
(b) Let $g(x)=f(2 x-1)+k$, where $k$ is a constant. The graph of $y=g(x)$ touches the $x$-axis at one point.
(i) Find the value of $k$.
(ii) If $\alpha$ and $\beta$ are the roots of the quadratic equation $g(x)=f(x)$, find a quadratic equation in $y$ with roots are $\alpha^{2}$ and $\beta^{2}$.

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Answers written in the margins will not be marked

12. In Figure 3, two circles $A B F$ and $D E F$ touch each other. $C H F$ is the common tangent to the two circles at $F$. $A F E, A B C, C D E$ and $B H D$ are straight lines. Let $\angle C A E=p$ and $\angle C E A=q$.


Figure 3
(a) (i) Prove that $B, C, D$ and $F$ are concyclic.
(ii) Hence, prove that $A, B, D$ and $E$ are concyclic.
(b) Suppose $A F$ and $F E$ are diameters of the circles $A B F$ and $D E F$ respectively.

David claims that the orthocentres of $\triangle C F A$ and $\triangle C F E$ lie on the same point. Do you agree? Explain your answer.
(2 marks)

13. Solve the equation $\log _{2}(x-1)+\log _{2}(2 x+3)=3$, leave your answers in surd form. (4 marks) $\ldots+$.

