## 19-20 F. 5 1st TERM EXAM-MATH-CP 1

19-20 F. 5 1st TERM EXAM MATH CP PAPER 1

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2019-2020
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Form 5 First Term Examination

## MATHEMATICS Compulsory Part

## PAPER 1

## Question-Answer Book

$2^{\text {nd }}$ January, 2020
8:15 am - 10:00 am (1 hour 45 minutes)
This paper must be answered in English

## INSTRUCTIONS

1. Write your name, class and class number in the spaces provided on this cover.
2. This paper consists of THREE sections, $\mathrm{A}(1)$, $\mathrm{A}(2)$ and B .
3. Attempt ALL questions in this paper. Write your answers in the spaces provided in this Question - Answer Book. Do not write in the margins. Answers written in the margins will not be marked.
4. Unless otherwise specified, all working must be clearly shown.
5. Unless otherwise specified, numerical answers should be either exact or correct to 3 significant figures.
6. The diagrams in this paper are not necessarily drawn to scale. daw

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| Sections | Marks |
| :---: | :--- |
| $\mathrm{A}(1-4)$ |  |
| $\mathrm{A}(5-12)$ |  |
| A Total | $/ \mathbf{/ 5 6}$ |
| B Total | $/ \mathbf{8 4}$ |
| TOTAL |  |

## Section A(1) (28 marks)

1. Simplify $\frac{x^{-2}\left(x^{3} y^{5} z^{0}\right)^{4}}{y^{25}}$ and express your answer with positive indices.
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2. The radius and the arc length of a sector are 20 cm and $9 \pi \mathrm{~cm}$ respectively.
(a) Find the angle subtended by the sector at the centre.
(b) Express the area of the sector in terms of $\pi$.
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3. Factorize
(a) $5 x^{2}-6 x y+y^{2}$,
(b) $5 x^{2}-6 x y+y^{2}-15 x+3 y$.
$\qquad$
 $\frac{4}{4}+\frac{1}{4}+\frac{1}{4}$
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Answers written in the margins will not be marked
4. In the figure, the stem-and-leaf diagram shows the weights of 15 students in a class. If the mean weight and median weight are 45.6 kg and 44 kg respectively, find
(a) $x$ and $y$;
(b) the mode weight.
(4 marks)
The weight of 15 students in a class
$\qquad$

| Stem $(10 \mathrm{~kg})$ | Leaf( 1 kg$)$ |
| ---: | :--- |
| 3 | 356 |
| 4 | $0 x x 4 y 9$ |
| 5 | 11233 |
| 6 | 3 |

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5. Consider the compound inequality

$$
\begin{equation*}
16-8 x \geq 0 \text { and } \frac{15-2 x}{7}<12+3 x \tag{*}
\end{equation*}
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(a) Solve (*).
(b) Write down the least integer satisfying the compound inequality in (a).
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Answers written in the margins will not be marked
6. In the figure, $O$ is the centre of the circle. $A, B, C$ and $D$ are points lying on the circle. It is given that $\angle A B O=32^{\circ}, \angle A D C=112^{\circ}$ and $\angle D C O=44^{\circ}$. Find $\angle O B C$ and $\angle B A D$.
(4 marks)

7. The figure shows some water in a cylinder of base radius 4 cm and height 8 cm . The water level is 5 cm high. A number of identical spherical marbles each of radius 2 cm are put into the cylinder. Assuming that all the marbles are completely immersed into the water, find the maximum number of marbles that can be put into the cylinder so that the water will not overflow.


## Section A(2) (28 marks)

8. Consider the equation $k x^{2}+(2 k+3) x+2+k=0$, where $k \neq 0$. Find the range of the values of $k$ if the equation has two distinct real roots.
9. The following box-and-whisker diagram shows the distribution of weights (in kg ) of 35 students. The mean weight of the students is 64 kg .

(a) Find the range and the inter-quartile range of the distribution.
(3 marks)
(b) Five students are joined and their weights are $55 \mathrm{~kg}, 62 \mathrm{~kg}, 63 \mathrm{~kg}, 68 \mathrm{~kg}$ and 76 kg . Find the new mean and the new median of the weight.
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Answers written in the margins will not be marked
10. (a) Let $f(x)=24 x-x^{2}$. Find the coordinates of the vertex of the graph of $y=f(x)$ by using the method of completing the square.
(b) There is a piece of string of length 120 m . David cuts the string into 4 pieces such that the longest one is used to enclose a rectangular zone of area $A \mathrm{~m}^{2}$. The other 3 pieces are of the same length of $x \mathrm{~m}$ which are used to divide the zone into 4 parts as shown in the figure.
(i) Express $A$ in terms of $x$.
(ii) David claims that the area of the rectangular zone cannot be greater than $300 \mathrm{~m}^{2}$. Do you agree? Explain your answer.

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11. Let $\$ S$ be the selling price of a cup of coffee with capacity $V \mathrm{~mL}$. It is given that $S$ is the sum of two parts, one part varies directly as $V$ and the other part varies directly as the square of $V$. When $V=500, S=35$; when $V=700, S=63$.
(a) Find the selling price of a cup of coffee with capacity 250 mL .
(b) There is a larger cup of coffee and the selling price of the larger cup is 4 times that of the cup described in (a). Find the capacity of the larger cup of coffee.
12. The coordinates of $H$ and $K$ are $(3,2)$ and $(11,8)$ respectively. Let $C$ be the circle with $H K$ as its diameter.
(a) Find the equation of $C$.
(b) $P$ is a moving point in the coordinate plane such that $H P=K P$. Denote the locus of $P$ as $\Gamma$.
(i) Find the equation of $\Gamma$.
(ii) Describe the geometric relation between $H K$ and $\Gamma$.
(iii) Suppose that $\Gamma$ intersects $C$ at $M$ and $N$. Find the area of the quadrilateral $H M K N$.

## Section B (28 marks)

13. The table below shows the means and the standard deviations of the time for a large group of students to finish a 100 m run in two fitness tests:

| Test | Mean | Standard deviation |
| :---: | :---: | :---: |
| I | 20 s | 2 s |
| II | 18 s | 1 s |

The standard score of the time for Billy to finish the run in test I is 1.5 .
(a) Find the time for Billy to finish the run in test I.
(b) Assume that the time distribution in each of the above tests are normally distributed. The time for Billy to finish the run in test II is 20 s . He claims that comparing to other students, he performs better in test II than that in test I. Is his claim correct? Explain your answer.

Answers written in the margins will not be marked
14. In the figure, the graph shows a linear relation between $x$ and $\log _{4} y$. The slope and the intercept on the vertical axis of the graph are -2 and $\frac{5}{2}$ respectively. Express the relation between $x$ and $y$ in the form $y=A k^{x}$, where $A$ and $k$ are constants.


Answers written in the margins will not be marked
15. $A(7,6)$ is a point lies on the circle $C$ with the centre $(1,0)$.
(a) Find the equation of $C$.
(b) Show that circle $C$ and straight line $L: y=x+11$ have only 1 point of intersection.
16. In the figure, the equations of $L_{1}$ and $L_{2}$ are $x=60$ and $y=10$ respectively. The slope of the straight line $L_{3}$ is $\frac{1}{3}$. The straight line $L_{4}$ intersects $L_{2}$ and $L_{3}$ at $(380,10)$ and $(240,80)$ respectively.

(a) (i) Find the equations of $L_{3}$ and $L_{4}$.
(ii) In the figure above, the shaded region (including the boundary) represents the solution of a system of inequalities. Write down the system of inequalities. (4 marks)
(b) An engineer wants to build an aeroplane which consists of two classes: economy class and first class. It is given that the aeroplane must have at least 60 economy class seats and 10 first class seats. Moreover, the number of economy class seats in the aeroplane must not less than 3 times that of the first class seats. Each economy class seat occupies a floor area of $10 \mathrm{~m}^{2}$ and each first class seat occupies a floor area of $20 \mathrm{~m}^{2}$. The floor area occupied by the seats in the aeroplane is at most $4000 \mathrm{~m}^{2}$. The aeroplane is used to fly a certain flight. On that flight, the profits of selling an economy class ticket and a first class ticket are $\$ 4000$ and $\$ 15000$ respectively. The manager of the airline claims that if all the tickets of the flight are sold, the total profit is not more than $\$ 2160000$. Do you agree? Explain your answer.

## (16 cont.)

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Answers written in the margins will not be marked
17. It is given that $f(x)=x(x-5)(2 x+3)$. The H.C.F. and L.C.M. of $f(x)$ and $g(x)$ are $(x-5)(2 x+3)$ and $4 x(x+1)(x-5)(2 x+3)^{2}$ respectively.
(a) Find $g(x)$.
(2 marks)
(b) When $g(x)$ is divided by $h(x)$, the quotient and the remainder are both $x+5$. Find the remainder when $h(x)$ is divided by $x$.
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## End of Paper

Answers written in the margins will not be marked

