

18-19 F. 5 1st TERM EXAM MATH CP PAPER 1

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2018-2019
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Form 5 First Term Examination

## MATHEMATICS Compulsory Part

## PAPER 1

## Question-Answer Book

$2^{\text {nd }}$ January, 2019
8:15 am - 10:00 am (1 hour 45 minutes) This paper must be answered in English

## INSTRUCTIONS

1. Write your name, class and class number in the spaces provided on this cover.
2. This paper consists of THREE sections, $\mathrm{A}(1)$, $\mathrm{A}(2)$ and B .
3. Attempt ALL questions in this paper. Write your answers in the spaces provided in this Question - Answer Book. Do not write in the margins. Answers written in the margins will not be marked.
4. Unless otherwise specified, all working must be clearly shown.
5. Unless otherwise specified, numerical answers should be either exact or correct to 3 significant figures.
6. The diagrams in this paper are not necessarily drawn to scale. drawne


| Sections | Marks |
| :---: | ---: |
| $\mathrm{A}(1-4)$ |  |
| $\mathrm{A}(5-11)$ |  |
| A Total | $/ \mathbf{5 6}$ |
| B Total | $/ \mathbf{8 4}$ |
| TOTAL |  |

## Section A(1) (28 marks)

1. Simplify $\frac{x^{5} y^{-2}}{\left(x^{2} y^{-3}\right)^{-2}}$ and express your answer with positive indices.
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2. Factorize
(a) $4 p-10 q$,
(b) $2 p^{2}+p q-15 q^{2}$,
(c) $4 p-10 q-2 p^{2}-p q+15 q^{2}$.
3. Consider the formula $x+8=3(5 x+2 y)$.
(a) Make $y$ the subject of the above formula.
(b) If the value of $x$ is decreased by 6 , find the change in the value of $y$.
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$\qquad$ (4)

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Answers written in the margins will not be marked
4. The marked price of a toy is $\$ 144$. It is given that the marked price of the toy is $20 \%$ higher than its cost.
(a) Find the cost of the toy.
(b) If the toy is sold at a discount of $20 \%$ on its marked price, determine whether there will be a gain or a loss on selling the toy. Explain your answer.
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5. Consider the compound inequality

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\begin{equation*}
-6 x \geq 12 \quad \text { and } \quad \frac{2 x+5}{3}>2(x+1) \tag{*}
\end{equation*}
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(a) Solve (*).
(b) Write down the greatest negative integer satisfying (*).
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6. The coordinates of the points $P$ and $Q$ are $(-1,-4)$ and $(a,-2)$ respectively. $P$ is rotated clockwise about the origin $O$ through $90^{\circ}$ to $P^{\prime}$.
(a) Find the coordinates of $P^{\prime}$.
(b) If $P^{\prime}, O$ and $Q$ are collinear, find $a$.
7. Let $g(x)=x^{3}-x^{2}-11 x+c$, where $c$ is a constant. When $g(x)$ is divided by $x+1$, the remainder is 12 .
(a) Is $x+3$ a factor of $g(x)$ ? Explain your answer.
(b) David claims that all the roots of the equation $g(x)=0$ are rational. Do you agree? Explain your answer.
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## Section A(2) (28 marks)

8. The stem-and-leaf diagram below shows the ages of the players of a football team:

| Stem (tens) | Leaf (units) |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 1 | 8 | 9 | 9 |  |  |  |  |  |  |  |  |  |  |  |  |
| 2 | 0 | 1 | 1 | 1 | 3 | 3 | 5 | 6 | 6 | 7 | 7 | 8 | 8 | 8 |  |
| 3 |  |  | 1 |  |  |  |  |  |  |  |  |  |  |  |  |

(a) Find the mean, the median and the range of the ages of the players of the football team.
(b) As the two oldest players leave the team, three new players join the football team. After the three players join the football team, the manager of the team finds the mean age of the players of the football team is the same as the mean found in (a).
(i) Find the mean age of the three new players.
(ii) Furthermore, the manager finds that the median and the range of the ages of the players of the football team are the same as the median and the range found in (a) respectively. Write down two sets of possible ages of the three new players.
(4 marks)
9. In Figure 1, $B D$ is a diameter of the circle, $D E=5 \sqrt{3} \mathrm{~cm}, C D=10 \sqrt{3} \mathrm{~cm}, A B=B E, A D C$ and $B E C$ are straight lines.
(a) Prove that $\triangle A B D \cong \triangle E B D$.
(3 marks)
(b) Prove that $\triangle A B C \sim \triangle E D C$.
(2 marks)
(c) Find $A B$.


Figure 1
10. Jack wants to rent a new apartment. The rent $(\$ R)$ per month of an apartment consists of two parts. One part is a constant and the other part varies directly as the size ( $S$ sq.ft.) of the apartment and varies inversely as the time ( $T$ months) that he rents an apartment. If he rents an apartment of 800 sq . ft. for 6 months, the rent is $\$ 25000$ per month. The rent of an apartment of 1000 sq. ft. for 24 months is $\$ 20875$ per month.
(a) Find the rent per month of an apartment of 600 sq . ft. for 36 months.
(4 marks)
(b) John claims that the rent of two 500 sq . ft. apartments for 24 months is cheaper than the rent of a 1000 sq . ft. apartment for the same period of time. Do you agree? Explain your answer.
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Answers written in the margins will not be marked
11. The circle $C$ passes through the point $A(4,-3)$ and the centre of $C$ is the point $G(2,0)$.
(a) Find the equation of $C$. (2 marks)
(b) $P$ is a moving point in the rectangular coordinate plane such that $A P=G P$. Denote the locus of $P$ by $\Gamma$.
(i) Find the equation of $\Gamma$.
(ii) Describe the geometric relationship between $\Gamma$ and the line segment $A G$.
(iii) If $\Gamma$ cuts $C$ at $Q$ and $R$, find the perimeter of the quadrilateral $A Q G R$.

## Section B (28 marks)

12. The standard deviation of the scores of a class of students in an examination is 4 marks. The score of Joey and Andy in the examination are 58 marks and 86 marks respectively. The standard score of Joey in the examination is -2 .
(a) Find the standard score of Andy in the examination.
(b) A student, Bobby, is in the class and his teacher forgot to insert his examination score. It is given that his examination score is 66 . Will there be any change in the standard score of Andy due to the insertion of the examination score of Bobby? Explain your answer.
13. In Figure 2, $C$ is the centre of the circle $P Q S . O R$ and $O P$ are tangent to the circle at $S$ and $P$ respectively. $O C Q$ is a straight line and $\angle Q O P=30^{\circ}$.

(a) Show that $\angle P Q O=30^{\circ}$.
(b) Suppose $O P Q R$ is a cyclic quadrilateral.
(i) Show that $R Q$ is tangent to circle $P Q S$ at $Q$.
(ii) A rectangular coordinate system is introduced in Figure 2 so that the coordinates of $O$ and $C$ are $(0,0)$ and $(6,8)$ respectively. Find the equation of $Q R$.
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14. (a) In Figure 3, $L_{1}$ and $L_{2}$ intersect at the point $(9,2)$ while $L_{2}$ and $L_{3}$ intersect at the point $(6,5)$. The equation of $L_{3}$ is $5 x+6 y-60=0$.


Figure 3
(i) Find the equations of $L_{1}$ and $L_{2}$.
(ii) In Figure 3, the shaded region (including the boundary) represents the solution of a system of inequalities. Write down the system of inequalities.
(b) The owner of a boat-building yard decided to build two types of boats, type $A$ and type $B$, under the following conditions.
(1) A boat of type $A$ needs $30 \mathrm{~m}^{2}$ of working space and a boat of type $B$ needs $36 \mathrm{~m}^{2}$ of working space. He has $360 \mathrm{~m}^{2}$ of working space.
(2) The expenses on materials for building a boat of type $A$ and a boat of type $B$ are $\$ 8000$ and $\$ 4000$ respectively. He can spend at most $\$ 80000$ on buying the materials.
(3) He has enough manpower to build at most 11 boats.

Let $x$ and $y$ be the numbers of boats of type $A$ and type $B$ to be built respectively. Suppose the profit on a boat of type $A$ is $20 \%$ more than that on a boat of type $B$, Find the number of boats of each type that should be built to maximize the total profit on selling the two types of boats.
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15. (a) Express $\frac{1}{4+3 i}$ in the form of $a+b i$, where $a$ and $b$ are real numbers. (1 mark)
(b) The roots of the quadratic equation $x^{2}+p x+q=0$ are $\frac{25}{4+3 i}$ and $\frac{25}{4-3 i}$. Find $p$ and $q$.

