## 2017-2018 F. 5 2nd TERM EXAM-MATH-CP 1

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2017-2018
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Form $52^{\text {nd }}$ Term Examination

## MATHEMATICS Compulsory Part

 PAPER 1
## Question-Answer Book

$8^{\text {th }}$ June, 2018
8:15 a.m. - 10:30 a.m. (2 hours 15 minutes) This paper must be answered in English.

## INSTRUCTIONS

1. Write your name, class and class number in the spaces provided on this cover.
2. This paper consists of THREE sections, $\mathrm{A}(1)$, $\mathrm{A}(2)$ and B .
3. Attempt ALL questions in this paper. Write your answers in the spaces provided in this Question-Answer Book. Do not write in the margins. Answers written in the margins will not be marked.
4. Unless otherwise specified, all working must be clearly shown.
5. Unless otherwise specified, numerical answers should be either exact or correct to 3 significant figures.
6. The diagrams in this paper are not necessarily drawn to scale.


| Section | Marks |
| :---: | :---: |
| $\mathrm{A}(1-3)$ |  |
| $\mathrm{A}(4-14)$ | $/ \mathbf{7 0}$ |
| A Total | $/ \mathbf{3 5}$ |
| B Total |  |
| TOTAL | $\mathbf{1 0 5}$ |

Section A(1) (35 marks)

1. Simplify $\frac{\left(x y^{2}\right)^{-4}}{x^{3}}$ and express your answer with positive indices.
2. Make $y$ the subject of the formula $x(y-1)=2 y-5$.
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Answers written in the margins will not be marked.
3. Factorize
(a) $4 a^{2}-4 a b+b^{2}$,
(b) $1-4 a^{2}+4 a b-b^{2}$.
4. (a) Solve the inequality $\frac{4 x+1}{5}+3>x$.
(b) How many integers satisfy both the inequalities $\frac{4 x+1}{5}+3>x$ and $2 x+2 \geq 9$ ?
(4 marks)
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Answers written in the margins will not be marked.
5. The radius and the area of a sector are 10 cm and $2 \pi \mathrm{~cm}^{2}$ respectively.
(a) Find the angle at the centre of the sector.
(b) Express the perimeter of the sector in terms of $\pi$.
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6. (a) Prove that $\left(\frac{1}{\cos \theta}+\tan \theta\right)^{2}=\frac{1+\sin \theta}{1-\sin \theta}$.
(b) Hence, solve $\left(\frac{1}{\cos x}+\tan x\right)^{2}=2$, where $0^{\circ} \leq x \leq 90^{\circ}$.
(5 marks)
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Answers written in the margins will not be marked.
7. In the figure, the straight line $L: 4 x-3 y-36=0$ cuts the $x$-axis and the $y$-axis at points $A$ and $B$ respectively.

(a) Find the area of $\triangle A O B$.
(b) $C(6, k)$ is a point on the straight line $L$ where $k$ is a constant. Is $O C$ an altitude of $\triangle A O B$ ? Explain your answer.
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8. The table below shows the distribution of the ages of 50 members in a club.

| Age less than | Cumulative frequency |
| :---: | :---: |
| 60 | 50 |
| 50 | 38 |
| 40 | $x$ |
| 30 | $\frac{x}{2}$ |

It is given that the ratio of the number of the members aged from 50 to 59 and that of the members aged from 30 to 39 is $6: 5$.
(a) Find $x$.
(b) A member in the club is chosen at random. Find the probability that the age of the chosen member is between 40 and 49 .
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Answers written in the margins will not be marked.
9. In the figure, $O$ is the centre of the circle. $A D$ and $C E$ are diameters of the circle. It is given that $\angle O C B=64^{\circ}$ and $\angle O E D=34^{\circ}$.

(a) Find $\angle O A B$.
(b) Find $\frac{A B}{A D}$.
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## Section A(2) (35 marks)

10. The following box-and-whisker diagram shows the distribution of the weights of 60 dogs in a pet shop.

(a) What is the weight of the heaviest dog?
(b) Find the range and the inter-quartile range of the weights of the dogs.
(c) How many dogs that are less than 35 kg ?
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Answers written in the margins will not be marked.
11. Let $\$ C$ be the cost of making a toy car of volume $V \mathrm{~cm}^{3}$. It is given that $C$ is the sum of two parts, one part is a constant and the other part varies directly as $V$. When $V=40, C=25$; when $V=48, C=29$.
(a) Find the cost of making a toy car of volume $28 \mathrm{~cm}^{3}$.
(b) Jenny has a bigger toy car which is similar to the toy car described in (a). The surface area of the bigger toy car is 4 times that of the toy car described in (a). Jenny claims that the cost of making the bigger toy car is 8 times that of making the toy car in (a). Do you agree? Explain your answer.
12. Figure (a) shows a closed vessel with some water, which is in form of an inverted right circular cone. The base radius and the height of the cone are 18 cm and 24 cm respectively. Kelly finds that the depth of water in the vessel is 8 cm .


Figure (a)


Figure (b)
(a) (i) Find the radius of the water surface.
(ii) Find the area of the wet curved surface of the vessel in terms of $\pi$.
(b) Kelly turns the vessel upside down as shown in Figure (b).
(i) Find the new depth of water, correct to 1 decimal place.
(ii) Kelly claims that the wet curved surface of the vessel has increased. Do you agree? Explain your answer.
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13. The coordinates of points $X$ and $Y$ are $(-3,7)$ and $(5,1)$ respectively. $P$ is a moving point in the rectangular coordinate plane such that $P X=P Y$. Denote the locus of $P$ by $\Gamma$.
(a) (i) Find the equation of $\Gamma$.
(ii) Describe the geometric relationship between $\Gamma$ and the line segment $X Y$.
(4 marks)
(b) It is given that a circle passes through $X$ and $Y$. Denote the centre of the circle by $R$.
(i) Does $\Gamma$ pass through $R$ ? Explain your answer.
(ii) The line $y=x$ divides the circle into two equal halves. Find the coordinates of $R$.
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14. Let $f(x)=x^{3}+(k-2) x^{2}-3 x+1$, where $k$ is a positive constant. When $f(x)$ is divided by $x-1$ and $x+k$ respectively, the remainders are the same.
(a) Find the value of $k$.
(b) How many real roots are there for the equation $x^{3}+(k-2) x^{2}-3 x+1=(x-1)(2 x-5)-1$ ? Explain your answer.
Section B (35 marks)
15. A wallet contains three $\$ 10$ notes, five $\$ 20$ notes and four $\$ 100$ notes.
(a) If 4 notes are drawn at random at the same time, find the probability that the total value of the notes drawn is $\$ 150$.
(b) If 3 notes are drawn at random at the same time, find the probability that the total value of the notes remaining in the wallet is less than $\$ 50$.
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16. (a) Express $\frac{1}{3-i}$ in the form of $a+b i$, where $a$ and $b$ are real numbers.
(b) The roots of the quadratic equation $x^{2}+c x+d=0$ are $\frac{5}{3-i}$ and $\frac{5}{3+i}$.
(i) Find $c$ and $d$.
(ii) If the parabola $y=x^{2}+c x+d$ and the straight line $y=m x$ do not intersect, find the range of values of $m$.
17. Let $f(x)=-\frac{1}{5} x^{2}+8 x-77$.
(a) Using the method of completing the square, find the coordinates of the vertex of the graph of $y=f(x)$.
(2 marks)
(b) The graph of $y=g(x)$ is obtained by translating the graph of $y=f(x)$ vertically. It is known that the graph of $y=g(x)$ touches the $x$-axis.
(i) Write down $g(x)$.
(ii) Under a transformation, $g(x)$ is changed to $-\frac{1}{5} x^{2}-8 x-80$. Describe the geometric meaning of the transformation.
(3 marks)
18. In the figure, the circle $C$ passes through four points $Q, R, S$ and $T$, where $R T$ is a diameter of $C$. $R T$ cuts $Q S$ perpendicularly at $U$ and is produced to $P$ such that $P Q$ and $P S$ are tangents to $C$ at $Q$ and $S$ respectively.

(a) (i) Prove that $\triangle Q T U \cong \triangle S T U$.
(ii) Prove that $T$ is the in-centre of $\triangle P Q S$.
(b) A rectangular coordinate system is introduced on the figure so that the coordinates of $P, Q$ and $S$ are $(50, m),(0,48)$ and $(36,0)$ respectively. Denote the centre of $C$ by $H$.
(i) Find the coordinates of $T$.
(ii) Let $A$ be a point lying on the incircle of $\triangle P Q S$ such that $A$ is farthest to $H$. Find the distance between $A$ and $H$.
(7 marks)
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19. In the figure, the bearing of $B$ from $A$ is $120^{\circ}$ and the bearing of $C$ from $B$ is $230^{\circ}$. $A B=50 \mathrm{~m}$ and $B C=90 \mathrm{~m}$.

(a) Find the distance between $A$ and $C$.
(b) Find the shortest distance between $B$ and $A C$.
(c) A student claims that the shortest distance between $A$ and $B C$ is the same as the shortest distance between $B$ and $A C$. Do you agree? Explain your answer.
(2 marks)
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