**17-18 F.5**
**1st TERM EXAM**
**MATH CP**
**PAPER 1**

2017 – 2018
Form 5 First Term Examination

# MATHEMATICS Compulsory Part

## PAPER 1

## Question–Answer Book

4th January, 2018
8:15 am – 10:00 am (1 hour 45 minutes)
**This paper must be answered in English**

**INSTRUCTIONS**

1. Write your name, class and class number in the spaces provided on this cover.

2. This paper consists of THREE sections, A(1), A(2) and B.

3. Attempt ALL questions in this paper. Write your answers in the spaces provided in this Question – Answer Book. Do not write in the margins. Answers written in the margins will not be marked.

4. Unless otherwise specified, all working must be clearly shown.

5. Unless otherwise specified, numerical answers should be either exact or correct to 3 significant figures.

6. The diagrams in this paper are not necessarily drawn to scale.

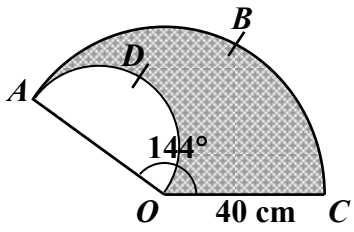
| Sections | Marks |
|---|---|
| A (1 – 4) | |
| A (5 – 11) | |
| **A Total** | **/57** |
| **B Total** | **/27** |
| **TOTAL** | **/84** |

Answers written in the margins will not be marked.

**Section A(1) (28 marks)**

**1.** Simplify $\dfrac{(y+y+y+y)^{-3}}{\left(x^{-2}y^{4}\right)^{3}}$ and express your answer with positive indices. (3 marks)

**2.** The figure below shows a sector *OABC* and a semicircle *OAD*. The radius of the sector *OABC* is 40 cm and the angle of the sector *OABC* is 144°. Find the perimeter of the shaded region. Give your answers in terms of $\pi$. (3 marks)




Answers written in the margins will not be marked.

**3.** Factorize

(a) $3p^2 - 2pq - 5q^2$,

(b) $3p^2 - 2pq - 5q^2 - p - q$.

(3 marks)

**4.** (a) Make $x$ the subject of the formula $\frac{1}{z} + \frac{2}{y} = \frac{5}{y} - \frac{3x}{2y}$.

(b) If $x : y : z = 2 : 5 : 3$, find the value(s) of $x$ satisfying the formula in (a).

(5 marks)



Answers written in the margins will not be marked.

Answers written in the margins will not be marked.

**5.** Consider the compound inequality

$$\frac{5-2x}{3} > 4(x+2) \quad \text{or} \quad 3x+11 \le 0 \qquad \text{…….. (*)}$$

**(a)** Solve (*).

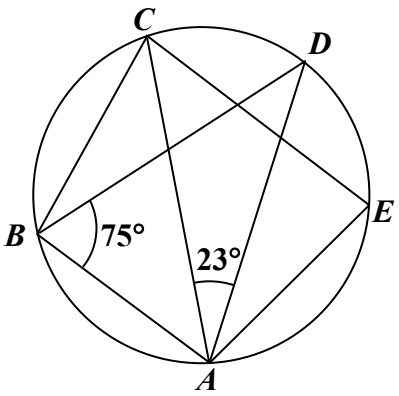
**(b)** Write down the greatest integer satisfying (*).

(4 marks)

Answers written in the margins will not be marked.

Answers written in the margins will not be marked.

**6.** In the figure, $ABCDE$ is a circle. It is given that $BA = BC$, $AB // EC$, $\angle CAD = 23°$ and $\angle ABD = 75°$. Find $\angle BAC$ and $\angle EAD$. (5 marks)






Answers written in the margins will not be marked.

Answers written in the margins will not be marked.

**7.** The $y$-intercept of the graph of $y = ax^2 + 11x + c$ is 12, where $a$ and $c$ are constants. It is given that the graph passes through (−6, 18).

**(a)** Find the values of $a$ and $c$. Hence, find the $x$-intercept(s) of the graph of $y = ax^2 + 11x + c$.

**(b)** It is given that $ax^2 + 11x + c = k$ has two distinct real roots, where $k$ is a constant. Find the range of values of $k$.

(5 marks)



Answers written in the margins will not be marked.

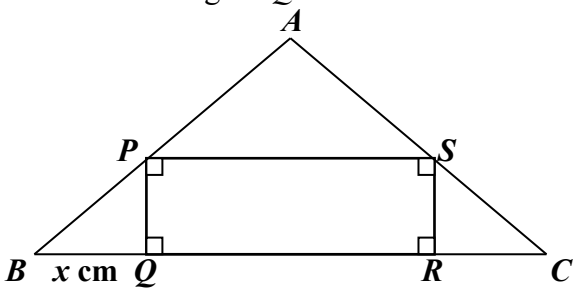
Answers written in the margins will not be marked.

**Section A(2) (29 marks)**

**8.** **(a)** Let $f(x) = 10x - x^2$. Find the coordinates of the vertex of the graph of $y = f(x)$.

(2 marks)

**(b)** In the figure, $\Delta ABC$ is a right-angled triangle with $AB = AC = 10\sqrt{2}$ cm and $BQ = x$ cm. Let $A$ cm$^2$ be the area of the rectangle $PQRS$.




(i) Express $A$ in terms of $x$.

(ii) Someone claims that $A$ cannot be greater than 60 cm$^2$. Do you agree? Explain your answer.

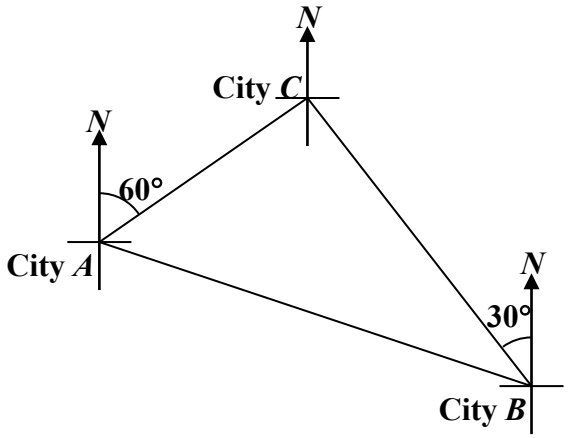
(4 marks)



Answers written in the margins will not be marked.

Answers written in the margins will not be marked.

**9.** In the figure, city $A$ and city $B$ are 50 km apart. At 8:00 a.m., Mr. Chan drives from city $A$ to city $C$ at a constant speed of 60 km/h in a direction of N60°E. At the same time, Mr. Lee drives from city $B$ to city $C$ at a constant speed of $x$ km/h in a direction of N30°W. They both arrive at city $C$ at 8:30 a.m..




**(a)** Find the true bearing of city $C$ from city $B$. (1 mark)

**(b)** Find $x$. (3 marks)

**(c)** Find the compass bearing of city $B$ from city $A$. (3 marks)

Answers written in the margins will not be marked.

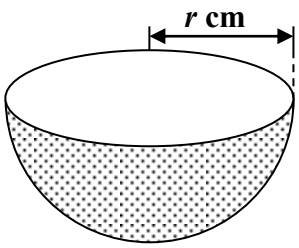
Answers written in the margins will not be marked.

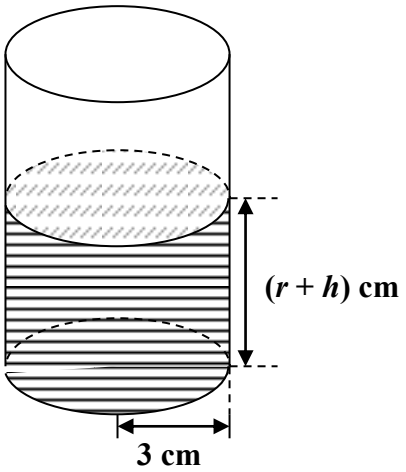
Answers written in the margins will not be marked.

Answers written in the margins will not be marked.

**10.** **(a)** Find the value of $k$ such that $2x-9$ is a factor of $8x^3-108x+k$. (2 marks)

**(b)** In the figure, a hemispherical container of radius $r$ cm is filled up with water. The water is poured into a cylindrical container of base radius 3 cm. Assume that the depth of the water in the cylindrical container is $(r + h)$ cm.




**(i)** Show that $2r^3-27r-27h=0$.

**(ii)** If $h=\frac{9}{4}$, find the value of $r$.

(5 marks)

Answers written in the margins will not be marked.

Answers written in the margins will not be marked.

Answers written in the margins will not be marked.

Answers written in the margins will not be marked.

**11.** The equation of the straight line $L_1$ is $3x + 4y = 0$.

**(a)** Let $L_2$ be the straight line passing through point $A(-7, 3)$ and perpendicular to $L_1$.

**(i)** Find the equation of $L_2$.

**(ii)** Suppose that $G$ is a point lying on $L_2$ different from $A$. Denote the $y$-coordinate of $G$ by $k$, where $k \neq 3$. Let $C$ be the circle passing through $A$ with centre $G$. Prove that the equation of $C$ is $2x^2 + 2y^2 - (3k - 37)x - 4ky + 143 - 9k = 0$.

(6 marks)

**(b)** The coordinates of the point $B$ are $(1, -1)$. Using **(a)(ii)**, or otherwise, find the radius of the circle which passes through $A$ and $B$ with centre $G$. (3 marks)



Answers written in the margins will not be marked.

Answers written in the margins will not be marked.

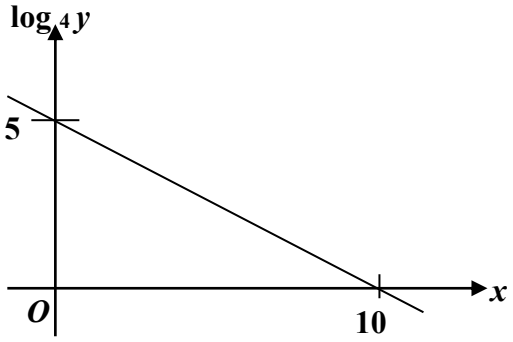
Answers written in the margins will not be marked.

Answers written in the margins will not be marked.

Answers written in the margins will not be marked.

**Section B (27 marks)**

**12.** The graph below shows the linear relation between $\log_4 y$ and $x$. The $x$-intercept and the intercept on the vertical axis of the graph are 10 and 5 respectively.




Express the relation between $x$ and $y$ in the form $y = Ak^x$, where $A$ and $k$ are constants.

(4 marks)



Answers written in the margins will not be marked.

Answers written in the margins will not be marked.

**13.** **(a)** Factorize $125x^3 - 8$. (2 marks)

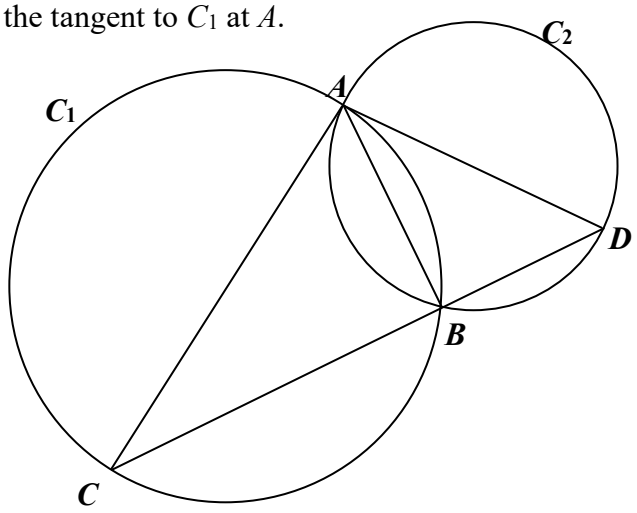
**(b)** Hence, solve the equation $125\sin^3\theta - 8 = 50\sin^2\theta + 20\sin\theta + 8$, where $0° \le \theta < 360°$. (4 marks)



Answers written in the margins will not be marked.

Answers written in the margins will not be marked.

**14.** In the figure, two circles $C_1$ and $C_2$ intersect at $A$ and $B$. $AC$ and $AD$ are diameters of $C_1$ and $C_2$ respectively. $AD$ is the tangent to $C_1$ at $A$.




(a) Prove that $AC$ is the tangent to $C_2$ at $A$. (2 marks)

(b) If $\angle ACB = 30°$ and $AC = 18$, find the radius of $C_2$. (5 marks)



Answers written in the margins will not be marked.

Answers written in the margins will not be marked.

Answers written in the margins will not be marked.

Answers written in the margins will not be marked.

Answers written in the margins will not be marked.

**15.** The nutritional values of a fish can and a meat can are shown in the table below.

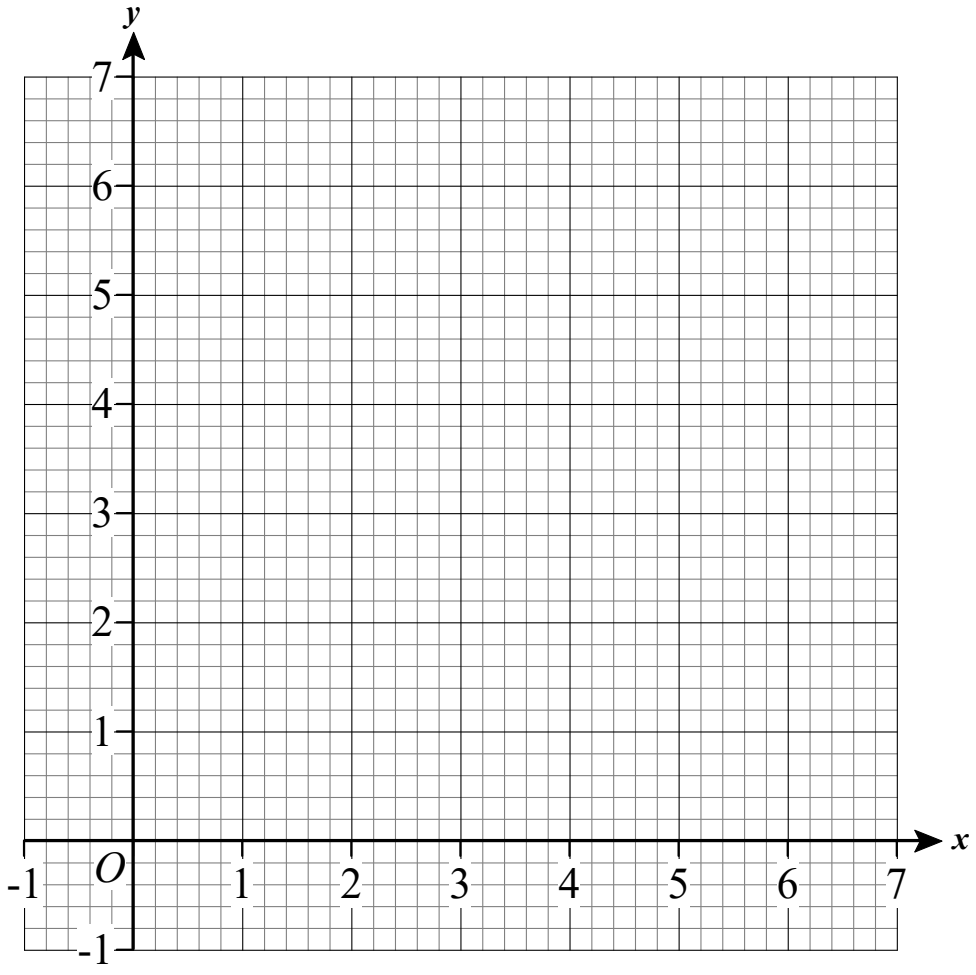
| | Protein(units) | Fat (units) | Iron (units) |
|---|---|---|---|
| Fish can | 2 | 1 | 1 |
| Meat can | 1 | 3 | 1 |

A dog needs a minimum intake of 5 units of protein, 6 units of fat and 4 units of iron per day. Let $x$ and $y$ be the numbers of fish cans and meat cans respectively for a dog per day.

**(a)** Write down the system of inequalities that represent the constraints on $x$ and $y$. Let $R$ be the region of the points representing the ordered pairs $(x, y)$ which satisfy these inequalities. Draw and shade the region $R$ in the figure below. (5 marks)

**(b)** It is given that the costs of a fish can and a meat can are \$3.5 and \$4 respectively, find the economical way and the minimum cost to provide the intake for the dog per day. (3 marks)

**(c)** If the cost of a fish can increases from \$3.5 to \$4.5, will the minimum cost to provide the intake increase by \$3 per day? Explain your answer. (2 marks)






Answers written in the margins will not be marked.

Answers written in the margins will not be marked.

Answers written in the margins will not be marked.

Answers written in the margins will not be marked.

Answers written in the margins will not be marked.

**End of Paper**

Answers written in the margins will not be marked.