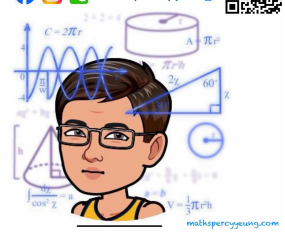

# Chapter 1 Laws of Indices

## Multiple Choice Section

**1.** $x^{2-6} =$

A. $\frac{x^2}{x^6}$ B. $\frac{x^6}{x^2}$ C. $x^2 \cdot x^6$ D. $x^{-2} \cdot x^6$

**2.** Simplify $(a^2 \cdot a^m)^3$.

A. $a^{6m}$ B. $a^{5+m}$ C. $a^{5+3m}$ D. $a^{6+3m}$

**3.** Simplify $(-\frac{1}{5}ab^2)^3$.

A. $\frac{1}{125}a^3b^6$ B. $-\frac{1}{125}a^3b^6$ C. $\frac{1}{15}a^2b^5$ D. $-\frac{1}{15}a^3b^5$

**4.** Simplify $\frac{(3x^2y)^3}{9x^5y^2}$.

A. $3x^4y$ B. $3x^3y$ C. $\frac{1}{3}xy$ D. $3xy$

**5.** Simplify $(\frac{2a}{3b^2})^3 \div (\frac{4a^3}{3b})^2$.

A. $\frac{1}{2a^2b^3}$ B. $\frac{1}{6a^3b^4}$ C. $\frac{1}{6a^2b^3}$ D. $\frac{128a^9}{243b^8}$

**6.** Simplify $3 \cdot 3^{n+3} - 3^{n+2}$.

A. $9 \cdot 3^{n+2}$ B. $8 \cdot 3^{n+3}$ C. $8 \cdot 3^{n+2}$ D. $24^{n+2}$

**7.** Simplify $\frac{4 \cdot 3^n + 9^n}{3^{n+2}}$.

A. 5 B. $\frac{13}{9}$ C. $\frac{4+n}{2}$ D. $\frac{4+3^n}{9}$

**8.** Simplify $\frac{s^s \cdot r^r}{s^r \cdot r^s}$.

A. $(sr)^{s-r}$ B. $(\frac{s}{r})^{r-s}$ C. $(\frac{s}{r})^{s-r}$ D. $4(sr)^{1-r}$

**9.** Which of the following is correct?

A. $a^0 \times a^7 = a^7$ B. $a^0 \times a^7 = a^0$ C. $a^0 \times a^7 = a^{-7}$ D. $a^0 \times a^7 = -7a$

**10.** Which of the following is incorrect?

A. $a^{m-n} = a^{-(n-m)}$ B. $a^{m-n} = \frac{a^m}{a^n}$ C. $a^{m-n} = \frac{a^n}{a^m}$ D. $a^{m-n} = \frac{1}{a^{n-m}}$

**11.** Simplify $(\frac{-ab^{-2}}{b^3})^0$.

A. 1 B. $-\frac{a}{b^5}$ C. $\frac{a}{b^5}$ D. $-\frac{a}{b}$

**12.** Simplify $(\frac{3}{4})^3 \div (\frac{2}{3})^{-2}$.

A. $\frac{16}{3}$ B. $\frac{2^8}{3^5}$ C. $\frac{3^5}{2^8}$ D. $\frac{3}{16}$

**13.** Simplify $[(x^{-2}y)^{-3}]^{-2}$ and express your answer in positive indices.

A. $\frac{y^6}{x^{12}}$ B. $\frac{x^{12}}{y^6}$ C. $x^{12}y^6$ D. $\frac{1}{x^{12}y^6}$

**14.** Simplify $(-\frac{7}{2}x^0y^{-1})^{-2}$.

A. $-\frac{4y^2}{49}$ B. $\frac{49}{4y^2}$ C. $-\frac{49}{4y^2}$ D. $\frac{4y^2}{49}$

**15.** Solve the exponential equation $3^{3x} = 729$.

A. 2 B. 3 C. 6 D. 27

**16.** Solve the exponential equation $11^x = 1$.

A. $\frac{1}{11}$ B. 0 C. 1 D. 11

**17.** Solve the exponential equation $7^x \times 7^{2x} = 1$.

A. $-\frac{1}{3}$ B. 0 C. $\frac{1}{3}$ D. 1

**18.** Solve the exponential equation $2^{x+3} + 2^x - 144 = 0$.

A. −4 B. 2 C. 3 D. 4

**19.** Express −40 in scientific notation.

A. $4^{-1}$ B. $4 \times 10^{-1}$ C. $-4 \times 10^1$ D. $-4 \times 10^{-1}$

**20.** Express $-4\,050 \times 10^{24}$ in scientific notation.

A. $4.05 \times 10^{-21}$ B. $4.05 \times 10^{21}$ C. $-4.05 \times 10^{26}$ D. $-4.05 \times 10^{27}$

**21.** Express $-0.001\,453 \times 10^{-21}$ in scientific notation.

A. $-1.453 \times 10^{-24}$ B. $-1.453 \times 10^{-23}$ C. $-1.453 \times 10^{-19}$ D. $-1.453 \times 10^{-18}$

**22.** Evaluate $\frac{1}{100} \times 10^{-3}$, express your answer in scientific notation and correct to 3 significant figures.

A. $1 \times 10^{-5}$ B. $1.00 \times 10^{-5}$ C. $0.001 \times 10^{-4}$ D. $0.01 \times 10^{-3}$

**23.** Evaluate $\sqrt{\frac{49 \times 10^9}{64 \times 10^{-9}}}$, express your answer in scientific notation and correct to 4 significant figures.

A. $8.750 \times 10^{17}$ B. $0.875 \times 10^9$ C. $8.750 \times 10^8$ D. $0.875 \times 10^0$

**24.** Evaluate $3.903 \times 10^{-4} - 4.03 \times 10^{-3}$, express your answer in scientific notation and correct to 4 significant figures.

A. $3.640 \times 10^{-3}$ B. $-3.640 \times 10^{-3}$ C. $3.500 \times 10^{-2}$ D. $-3.500 \times 10^{-2}$

**25.** Use all the numerals 1, 8 and 7 (without repetition) to form the greatest denary number.

A. 178 B. 718 C. 817 D. 871

**26.** Find the place value of the digit 7 in 1 070.

A. 1 B. 10 C. 100 D. 1 000

**27.** Express 3 094 in the expanded form with base 10.

A. $3 \times 10^3 + 9 \times 10^2 + 4$

B. $3 \times 10^3 + 9 \times 10^2 + 4 \times 10^0$

C. $3 \times 1\,000 + 9 \times 100 + 0 \times 10 + 4 \times 1$

D. $3 \times 10^3 + 9 \times 10^2 + 0 \times 10^1 + 4 \times 10^0$

**28.** If the digit 6 in 69 385 is replaced by digit 1, what is the difference between the numbers?

A. 50 B. 500 C. 5 000 D. 50 000

**29.** In 219 324, how many times is the place value of the left-most digit 2 to that of the right-most digit 2?

A. 10 000 times B. 1 000 times C. 100 times D. 10 times

**30.** Convert $1\ 101\ 101_2$ into a denary number.

A. 77 B. 93 C. 109 D. 218

**31.** Convert 25 into a binary number.

A. $10\ 011_2$ B. $11\ 001_2$ C. $11\ 011_2$ D. $11\ 111_2$

**32.** Which of the following binary numbers is even?

A. $10\ 110_2$ B. $10\ 111_2$ C. $10\ 101_2$ D. $11\ 111_2$

**33.** Arrange the following binary numbers in descending order.

$101\ 110_2$, $110\ 011_2$, $111\ 010_2$, $100\ 111_2$

A. $111\ 010_2 > 110\ 011_2 > 101\ 110_2 > 100\ 111_2$

B. $101\ 110_2 > 110\ 011_2 > 111\ 010_2 > 100\ 111_2$

C. $111\ 010_2 > 110\ 011_2 > 100\ 111_2 > 101\ 110_2$

D. $100\ 111_2 > 101\ 110_2 > 110\ 011_2 > 111\ 010_2$

**34.** Which of the following is not a hexadecimal number?

A. $E07_{16}$ B. $AB09_{16}$ C. $AC07_{16}$ D. $AK47_{16}$

**35.** Find the value of the digit F in $4CF2_{16}$.

A. 16 B. $16^2$ C. $15 \times 16$ D. $15 \times 16^2$

**36.** Convert $9CE_{16}$ into a denary number.

A. 2 496 B. 2 510 C. 39 936 D. 40 160

**37.** Convert 222 into a hexadecimal number.

A. $EF_{16}$ B. $DE_{16}$ C. $CD_{16}$ D. $BC_{16}$

## Section A(1)

**1.** If $a$ and $b$ are non-zero numbers, $m$ and $n$ are positive integers (where $m > n$), simplify the following expressions.

**(a)** $a^m \times a^n$ **(b)** $a^m \div a^n$ **(c)** $(a^m)^n$

**(d)** $(ab)^n$ **(e)** $(\frac{a}{b})^n$ **(f)** $a^0$

**2.** Simplify the following expressions.

**(a)** $(x^2)(x^6)$ **(b)** $x^{12} \div x^5$

**3.** Simplify the following expressions.

**(a)** $x^2 \cdot 2x^3 \cdot 3x^4$ **(b)** $(\frac{x^3}{y^2})^3$

**4.** Simplify the following expressions.

**(a)** $\frac{(3a^2)^3}{(a^3)^2}$ **(b)** $\frac{6a^9}{(3a^2)^2}$

**5.** Simplify the following expressions.

**(a)** $(x \cdot x^3)^2$ **(b)** $(\frac{x^3}{x^2})^3$

**6.** Simplify the following expressions.

**(a)** $(4x^2)^3$ **(b)** $\frac{(3xy)^3}{9x^2y}$

**7.** Evaluate the following without using a calculator.

**(a)** $2^{-2} \cdot 2^5$ **(b)** $\frac{1}{2^{-3}}$

**8.** Evaluate the following without using a calculator.

**(a)** $2^{-3} \div 2^0$ **(b)** $(3^{-1})^{-1}$

**9.** Evaluate the following without using a calculator.

**(a)** $(\frac{4}{3})^{-2}$ **(b)** $12^{-2} \div 2^{-4}$

**10.** Simplify the following expressions and express your answers in positive indices.

**(a)** $\frac{a^{-3}}{a^{-2}}$ **(b)** $\frac{ab^{-2}}{a^{-2}b^{-1}}$

**11.** Simplify the following expressions and express your answers in positive indices.

**(a)** $a^{-2} \times a^{-7}$ **(b)** $(-a^0b^{-2})^{-1}$

**12.** Simplify the following expressions and express your answers in positive indices.

**(a)** $(2x^{-1}y^2)^{-1}$ **(b)** $(x^{-2}y^{-3})^2$

**13.** Solve the following exponential equations.

**(a)** $2^x = 64$ **(b)** $3^x = \frac{1}{27}$

**14.** Solve the following exponential equations.

**(a)** $125^x = 1$ **(b)** $3^{2x} = 81$

**15.** If $5^{x-1} - 125 = 0$, find the value of $x$.

**16.** **(a)** Express 14 700 000 in scientific notation.

**(b)** Express the value of $1.414 \times 10^{-3}$ as a decimal number.

**17.** **(a)** Express 0.000 102 5 in scientific notation.

**(b)** Express the value of $6.023 \times 10^5$ as an integer.

**18.** Evaluate $(1.53 \times 10^{-4}) \times (4 \times 10^2)$ without using a calculator and express your answer in scientific notation.

**19.** Evaluate $(3 \times 10^{-5}) \div (6 \times 10^7)$ without using a calculator and express your answer in scientific notation.

**20.** Evaluate $(4.8 \times 10^{-6}) \div (0.3 \times 10^{-7}) \times (2 \times 10^{15})$ without using a calculator and express your answer in scientific notation.

**21.** Evaluate $\frac{0.000000000021 \div 70\,000000000}{0.0000000006}$ without using a calculator and express your answer in scientific notation.

**22.** Write down the place value of each underlined digit in the following denary numbers.

**(a)** 7$\underline{2}$3 **(b)** 4 $\underline{0}$71 **(c)** 30$\underline{0}$

**23.** Express the following denary numbers in expanded form with base 10.

**(a)** 34 **(b)** 598 **(c)** 1 024

**24.** Express the following as denary numbers.

**(a)** $4 \times 10^3 + 2 \times 10^2 + 0 \times 10 + 3 \times 1$

**(b)** $5 \times 10^4 + 0 \times 10^3 + 7 \times 10^2 + 3 \times 10 + 0 \times 1$

**25.** Write down the place value of the digit 0 in each of the following binary numbers.

**(a)** $1101_2$ **(b)** $11011_2$ **(c)** $10111_2$

**26.** Express the following binary numbers in the expanded form with base 2.

**(a)** $101_2$ **(b)** $11011_2$

**27.** Convert the following denary numbers into binary numbers.

**(a)** 27 **(b)** 42

**28.** Convert the following binary numbers into denary numbers.

**(a)** $101_2$ **(b)** $1101_2$

**29.** Write down the place value of the digit A in each of the following hexadecimal numbers.

**(a)** $3A_{16}$ **(b)** $A1B_{16}$

**30.** Express the following hexadecimal numbers in the expanded form with base 16.

**(a)** $A2_{16}$ **(b)** $2BF_{16}$

**31.** Convert the following hexadecimal numbers into denary numbers.

**(a)** $AF_{16}$ **(b)** $300_{16}$

**32.** Convert the following denary numbers into hexadecimal numbers.

**(a)** 31 **(b)** 168

## Section A(2)

**33.** Simplify the following expressions.

**(a)** $x(2x^2)(3x^3)$ **(b)** $\dfrac{(4a)(3a)^2}{(2a)^3}$

**34.** Simplify the following expressions.

**(a)** $(x^2y^3)^4(xyz)^2$ **(b)** $\dfrac{(xy^2)^3}{x^4y^5}$

**35.** Simplify the following expressions.

**(a)** $(\dfrac{y}{x^2})^3(\dfrac{x}{y^2})^2$ **(b)** $(\dfrac{a}{b^2})^2(\dfrac{2b}{a})^3$

**36.** Simplify the following expressions.

**(a)** $\dfrac{(3a^3b^4)^2}{a^6b^3}$ **(b)** $(\dfrac{3p^2q}{4pq})^3$

**37.** Simplify the following expressions and express your answers in positive indices.

**(a)** $\dfrac{[(4a^2)(3a)]^2}{a^3}$ **(b)** $\dfrac{(2ab)^2(a^2b)(bc)^2}{a^3b^3c^3}$

**38.** Simplify $\dfrac{(2x^2y^3)^5}{(-3xy^2)^2(2x)^2}$ and express your answer in positive indices.

**39.** Simplify the following expressions.

**(a)** $\dfrac{3^{n+1}}{3^{n-1}}$ **(b)** $\dfrac{3^{n+1}}{27^n}$

**40.** Simplify the following expressions.

**(a)** $\dfrac{3\cdot 9^n}{3^{n+1}}$ **(b)** $\dfrac{4^{n+1}\cdot 2^{n-1}}{2^{3n+1}}$

**41.** Simplify the following expressions.

**(a)** $\dfrac{4^{n+1}-4^{n-1}}{4^n}$ **(b)** $\dfrac{2(3^n)-3^{n-1}}{2(3^{n-1})}$

**42.** Simplify the following expressions and express your answers in positive indices.

**(a)** $\dfrac{x\cdot x^{-1}}{x^2}$ **(b)** $(\dfrac{2x^3y^2}{x^0y^3})^{-2}$

**43.** Simplify the following expressions and express your answers in positive indices.

**(a)** $(-2x^0y^2)(\dfrac{1}{2}x^3y^{-2})$ **(b)** $(-2x^4y^{-3})\div(x^{-4}y^{-2})$

**44.** Simplify $\dfrac{(2y^2)^{-3}\times 3(x^2)^{-1}}{(2y^{-1})^{-3}x^{-2}}$ and express your answer in positive indices.

**45.** Simplify $(\dfrac{-5x^2y}{z})^3 \div (225x^3y^2)^2$ and express your answer in positive indices.

**46.** Simplify $(\dfrac{729y^3}{125x^6z^9})^{\frac{1}{3}}$ and express your answer in positive indices.

**47.** If $x^{-\frac{1}{2}}=2$, find the value of $x$.

**48.** If $3^{2x+3}=9^{5-x}$, find the value of $x$.

**49.** If $7^{x-1}=1$, find the value of $x$.

**50.** If $(x^p)(x^{2p})=(x^3)^{\frac{1}{4}}$, find the value of $p$.

**51.** If $2^0+2^x+2^{-1}=1\dfrac{3}{4}$, find the value of $x$.

**52.** If $27^x=\dfrac{1}{9}$, find the value of $x$.

**53.** If $(8^{1+x})(2^{2x})=16^{2x}$, find the value of $x$.

**54.** If $3^{x+1}-3^x-162=0$, find the value of $x$.

**55.** If $5^{x+1}+5^{x-1}-650=0$, find the value of $x$.

**56.** It is given that the weight of one million oxygen atoms is $5.31\times10^{-17}$ g.

**(a)** Find the weight of one oxygen atom.

**(b)** Find the weight of a certain volume of gas which contains $2\times10^{13}$ oxygen atoms only.

(Express your answers in scientific notation.)

**57.** Express the following numbers in scientific notation (correct your answers to 3 significant figures) and arrange them in ascending order.

$13^{14}, 14^{13}, 12^{15}, 15^{12}$

**58.** **(a)** Use all the numerals 0, 1 and 2 (without repetition) to form all possible 3-digit denary numbers.

**(b)** Hence, write down the smallest 3-digit denary number.

**59.** **(a)** **(i)** Write down the smallest 4-digit denary number.

**(ii)** Write down the greatest 4-digit denary number.

**(b)** Hence, find the difference between the smallest and the largest 4-digit denary numbers.

## Section B

**60.** **(a)** If $2^{x+y}=16$, express $x$ in terms of $y$.

**(b)** Hence, if $\begin{cases}2^{x+y}=16\\2^{x-y}=2\end{cases}$, find the values of $x$ and $y$.

**61. (a)** Convert the following binary numbers into hexadecimal numbers.

| ***Binary number*** | $0000_2$ | $0001_2$ | $0010_2$ | $0011_2$ | $0100_2$ | $0101_2$ | $0110_2$ | $0111_2$ |
|---|---|---|---|---|---|---|---|---|
| ***Hexadecimal number*** | | | | | | | | |

| ***Binary number*** | $1000_2$ | $1001_2$ | $1010_2$ | $1011_2$ | $1100_2$ | $1101_2$ | $1110_2$ | $1111_2$ |
|---|---|---|---|---|---|---|---|---|
| ***Hexadecimal number*** | | | | | | | | |

**(b)** According to the results of **(a)**, convert the following binary numbers into hexadecimal numbers.

**(i)** $10101111_2$ **(ii)** $00011011_2$

**(c)** According to the results of **(a)**, convert the following hexadecimal numbers into binary numbers.

**(i)** $3A_{16}$ **(ii)** $C2D_{16}$